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On the neoricardian interpretation of profit

by
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One of the important components of the modern neoricardian theory is its critique of the marxian theory. A central place in this critique is occupied by the so-called surplus-approach which constitutes the modern neoricardian theory of profit.

As it is known, Marx interprets profit as the form of appearance of surplus value. Thus, according to him, surplus value is the sourse of profit. A positivistic reading of this fundamental marxian thesis says that the existence of positive profits is due to the existence of positive surplus value. It is against this positivistic formulation of the marxian theory of profit that the surplus-approach turns its critique. According to the surplus-approach the basis for the existence of positive profits is the existence of a (semi)positive surplus product (Steedman 1975, 1976, 1977 and Garegnani 1977).

The rejection by the neoricardian economists of the marxian theory of profit is founded on the alleged proof provided by Steedman (Steedman 1975, 1976, 1977) of the existence of negative labour values of commodities. The crux of his argument is that, since in joint production systems, negative labour values of commodities appear, one cannot rule out the case in which surplus value is negative, while surplus product is (semi)positive and profits are positive. Thus rendering impossible the interpretation of profit as the form of appearance of surplus value.

We have shown elsewhere (Stamatis 1979, 1983) that, in joint production systems the labour values of commodities are positive, albeit not unambigously determined magnitudes. So Steedman's argument is false.

In this article we will examine whether the neoricardian argument, according to which profits are positive because surplus product is (semi) positive, holds. This argument which was originally put forth by Steedman (Steedman 1975, 1976, 1977), is what we call surplus-approach.

Let us consider a production system $[\bar{A}, X]$ which produces single commodities, where $\bar{A}$, $\bar{A} > 0$, is the input matrix of means of production and wage commodities per unit of produced commodity and X, X>0, is the vector of this system's gross product. For $\bar{A}$ it holds that

$$\bar{A} = A + d\ell \ . \tag{1}$$

Matrix A, A≥0, is the input matrix of means of production per unit of produced commodity, d, d≥0, is the column vector of the real wage rate and ℓ, $\ell>0$, is the row vector of labour inputs per unit of produced commodity. Thus matrix $d\ell$,

$$d\ell = \begin{bmatrix} d_1\ell_1 & d_1\ell_1 & \cdots & d_1\ell_1 \\ d_1\ell_1 & d_1\ell_1 & \cdots & d_1\ell_1 \\ \vdots & \vdots & \vdots & \vdots \\ d_1\ell_1 & d_1\ell_1 & \cdots & d_1\ell_1 \end{bmatrix} \geq 0 \ , \tag{2}$$

is the matrix of real wages per unit of produced commodity.

For convenience sake, we assume that A and consequently $\bar{A}$ are irreducible matrices. This means that the given production system produces only basic commodities and, as a consequence, only reproductive commodities[1].

We assume that

$$(0<)\lambda_m^{\bar{A}} < 1 \ , \tag{3}$$

where $\lambda_m^{\bar{A}}$ the maximum eigenvalue of $\bar{A}$. Condition (3) implies that the technique $[A,\ell]$, which is used by the given production system $[A,\ell,X]$ for the given real wage rate d satisfying (2), namely the technique $[\bar{A}]$, is able of producing for every exogenously given (semi)positive surplus product U, U≥0,

1. Basic (non-basic) are those commodities that enter (do not enter) directly or indirectly the production of all commodities, except that of the commodity "labour power". Reproductive (non-reproductive) are those commodities that enter (do not enter) directly or indirectly the production of all commodities including the reproduction of the commodity "labour power". A basic commodity is always reproductive. A reproductive commodity can be basic or non-basic. A non-reproductive commodity is always non-basic. A non-basic commodity can be reproductive or non-reproductive.

a respective positive gross product X, X>0,[2] i.e. it is surplus-productive.

Proof:

For the surplus product U it holds

$$U = X - \bar{A}X = (I - \bar{A})X \ . \tag{4}$$

Because of (3) and the irreducibility of $\bar{A}$, we have

$$(I - \bar{A})^{-1} > 0 \ . \tag{5}$$

So, it follows from (4), taking (5) into account, that

$$X = (I - \bar{A})^{-1} U \ . \tag{6}$$

It follows, from (6), taking (5) into account, that, when U≥0, then X>0, i.e. finally that

$$\left.\begin{array}{l} U \geq 0 \\ \lambda_M^{\bar{A}} < 1 \\ \bar{A} \text{ is irreducible} \end{array}\right\} \Rightarrow X > 0 \ .$$

However as it is shown directly from equation (4), it does not necessarily hold

2. From (1) and (2) it follows that $\bar{A} \geq A$. From $\bar{A} \geq A$, $\bar{A} \geq 0$ and (3) together with the irreducibility of A it follows that $(0<)\lambda_m^A < \lambda_m^{\bar{A}} < 1$. The relation $(0<)\lambda_m^A < 1$ implies that the technique [A,ℓ] is productive, that is, the production system, which uses it, is able of producing for every exogenously given (semi)positive net product a respective (semi)positive (and here, because of the irreducibility of A, a positive) gross product. The productiveness of the technique [A,ℓ], i.e. the validity of $(0<)\lambda_m^A < 1$ is obviously a necessary but not sufficient condition for the surplus-productiveness of the technique [$\bar{A}$], i.e. for the validity of (3). And the surplus-productiveness of technique [$\bar{A}$], i.e. the validity of (3), is a sufficient but not necessary condition for the productiveness of the technique [A,ℓ], i.e. for the validity of $(0<)\lambda_m^A < 1$.

$$\left.\begin{array}{l} X>0 \\ \lambda_M^{\bar{A}}<1 \\ \bar{A} \text{ is irreducible} \end{array}\right\} \Rightarrow U \geq 0 \;.$$

This means: When the technique $[\bar{A}]$ is surplus-productive and at the same time X>0, then it is not necessarily U≥0, i.e. the surplus product is not necessarily (semi)positive, but it could be either (semi)positive or it could also include, apart from positive or positive and zero, negative components.

When, under the above conditions (i.e. X>0, $\lambda_m^{\bar{A}}<1$ and irreducibility of $\bar{A}$) U≥0, i.e. when the surplus product is (semi)positive, then the production system $[\bar{A},X]$ is viable, that is, it can reproduce itself. And when, under the same conditions, the surplus product includes, apart from positive or positive and zero, negative components, then the production system, although the technique that it uses is surplus-productive, is not viable, that is, it can not reproduce itself. In order to be able to reproduce itself, it needs, from outside the system, at least those quantities of commodities, which are included in its surplus product as negative quantities. Whether a production system, which uses a surplus-productive technique, is viable or not, obviously depends on the composition of its gross product.

Here we are primarily interested in this last case, i.e. in the case where the production system uses a surplus-productive technique, but it is not a viable one.

We will show that, in this case, we always have

$$pU>0 \tag{7}$$

and

$$wU>0, \tag{8}$$

where p and w denote the price vector and the labor value vector respectively. Proposition (7) means that, in the given case, the profit pU will always be positive, although the surplus product U is not (semi)positive, but includes, apart from positive or positive and zero, negative components. And proposition (8) means that, in the given case, the surplus value wU is always positive, although the surplus product U is not (semi)positive, but includes, apart from positive or positive and zero, negative components.

Depending on whether wages are paid at the beginning or at the end of the production period, for prices p it holds respectively

$$p^* = (1+r^*)p^*\bar{A} \Rightarrow$$

$$p^*[I-(1+r^*)\bar{A}] = 0 \quad (9)$$

or

$$\begin{aligned} p^{**} &= p^{**}A + r^{**}p^{**}A + p^{**}dw(I-A) \Rightarrow \\ p^{**}(I-A) &= r^{**}p^{**}A + p^{**}dw(I-A) \Rightarrow \\ p^{**} &= r^{**}p^{**}A(I-A)^{-1} + p^{**}dw \Rightarrow \\ p^{**}(I-dw) &= r^{**}p^{**}A(I-A)^{-1} \Rightarrow \\ p^{**} &= r^{**}p^{**}A(I-A)^{-1}(I-dw)^{-1} \Rightarrow \\ p^{**}[I-r^{**}A(I-A)^{-1}(I-dw)^{-1}] &= 0 \Rightarrow \\ p^{**}(I-r^{**}C) &= 0, \end{aligned} \quad (9a)$$

with

$$C \,\hat{=}\, A(I-A)^{-1}(I-dw)^{-1} = A(I-\bar{A})^{-1}(>0),$$

$$(I-\bar{A})^{-1} = (I-A)^{-1}(I-dw)^{-1},$$

where r^* (r^{**}) is the general rate of profit and p^* (p^{**}) is the price vector in the case, in which wages are paid at the beginning (at the end) of the production period.

For the price pU of the surplus product U, i.e. for the profit, it holds

$$pU = p(I-\bar{A})X\,. \quad (10)$$

Suppose that wages are paid at the beginning of the production period. Then, from (10), taking (9) into account, we obtain

$$\begin{aligned} p^*U &= p^*(I-\bar{A})X \\ &= p^*X - p^*\bar{A}X \\ &= (1+r^*)p^*\bar{A}X - p^*\bar{A}X \end{aligned}$$

$$= p^* \bar{A} X + r^* p^* \bar{A} X - p^* \bar{A} X$$

$$= r^* p^* \bar{A} X \,. \tag{11}$$

Since $r^* = \frac{1-\lambda_m^{\bar{A}}}{\lambda_m^{\bar{A}}}$ and $(0<)\lambda_m^{\bar{A}}<1$ we get $r^* > 0$. As it can be seen from (9), p^* is the left eigenvector of $\bar{A}$ which corresponds to the maximum eigenvalue $\lambda_m^{\bar{A}}$ of $\bar{A}$. Consequently, $p^* > 0$. Since $\bar{A} \geq 0$ and $X > 0$, it follows from (11) that

$$p^* U (= r^* p^* \bar{A} X) > 0 \,. \tag{12}$$

Suppose now that wages are paid at the end of the production period. Then, from (10), taking (9a) and (1) into account, we obtain

$$p^{**} U = p^{**} (I - \bar{A}) X$$

$$= p^{**} X - p^{**} \bar{A} X$$

$$= r^{**} p^{**} C X - r^{**} p^{**} C \bar{A} X$$

$$= r^{**} p^{**} A (I - \bar{A})^{-1} X - r^{**} p^{**} A (I - A)^{-1} X$$

$$= r^{**} p^{**} A (I - \bar{A})^{-1} (I - \bar{A}) X$$

$$= r^{**} p^{**} A X \,. \tag{11a}$$

Since $r^{**} = \frac{1}{\lambda_m^C}$, where λ_m^C is the maximum eigenvalue of C, and $\lambda_m^C > 0$, it follows from (9a), that $r^{**} > 0$. From (9a) it follows also that p^{**} is the left eigenvector of C, wich corresponds to the above maximum eigenvalue λ_m^C of C. Consequently, $p^{**} > 0$. Since $A \geq 0$ and $X > 0$, it follows from (11a) that

$$p^{**} U (= r^{**} p^{**} A X) > 0 \,. \tag{12a}$$

Thus profits are always positive, i.e. inequality (7) holds in both modes of wage payment, although the surplus product contains negative components.

We still have to prove the validity of (8). For the labour values we have

$$w = \ell(I-A)^{-1}\,(>0) \tag{13}$$

Thus we obtain

$$\begin{aligned} wU &= \ell(I-A)^{-1}U = \\ &= \ell(I-A)^{-1}(I-\bar{A})X = \\ &= \ell(I-A)^{-1}(I-A-d\ell)X = \\ &= \ell X - \ell(I-A)^{-1}\,d\ell X = \\ &= \ell X - wd\,(\ell X) = \\ &= \ell X(1-wd). \end{aligned} \tag{14}$$

According to (14), when $wd<1$, then $wU>0$, where wd is obviously the value of a unit of labour power. We will show that the value wd of a unit of labour power is less than unity and so that the surplus value wU is positive.

Proof: We get for $(I-\bar{A})$

$$\begin{aligned} (I-\bar{A}) = (I-A-d\ell) &= [(I-A)-dw(I-A)] = \\ &= (I-dw)(I-A). \end{aligned}$$

Hence, we get for $(I-\bar{A})^{-1}$

$$(I-\bar{A})^{-1} = [(I-dw)(I-A)]^{-1} = (I-A)^{-1}(I-dw)^{-1}.$$

Because of $(0<)\lambda_m^A<1$, is $(I-A)^{-1}>0$. Hence, because of (5) and $(I-A)^{-1}>0$, is $(I-dw)^{-1}>0$. The relation $(I-dw)^{-1}>0$ implies

$$(0<)\,wd<1,$$

where wd is the maximum eigenvalue of the matrix dw[3].

Thus the surplus value wU is always positive, although the surplus product contains negative components.

From the above it follows that, when the production system uses a surplus-productive technique but is non-viable, then it can not be said, as the neoricardian economists maintain, that profit is positive because the surplus product is (semi)positive, for the surplus product is not (semi)positive, but

3. On the proof of this see Stamatis (1998).

contains negative components as well. On the contrary it can be said that profit is positive because surplus value is positive.

However, let us examine whether it can be said, in the favourable for the neoricardians case, namely in the case, in which the production system not only uses a surplus-productive technique but it is also viable (i.e. in the case, in which $X>0$ ad at the same time $U\geq0$), that profit is positive because surplus product is (semi)positive.

In this case, of course, propositions (7) and (8) hold all the more. However, even in this case it can not be said that *in general* profit is positive because surplus product is (semi)positive. It is certainly correct that in this case to the positive profit of the whole production system corresponds a (semi)positive surplus product of the production system as a whole, i.e. that in this case we have both $pU>0$ and $U\geq0$. Nonetheless, even in this case it can not be said that *generally* profit is positive because surplus product is (semi)positive. For anything that holds for the production system as a whole does not necessarily hold for every single sector of this system. Each sector's profit is in this case always positive but the surplus product of each sector always contains negative components.

Demonstration:

Let $X_j s_j$,

$$X_j s_j = (0, 0, ..., X_j, ..., 0, 0)^T,$$

be the gross product of sector j, $j=1,2,...,n$, where X_j is that quantity of commodity j produced by sector j, with which this commodity is represented in the gross product X of the whole production system, and s_j is a nX1 column vector, all components of which, except the j-th which is equal to 1, are equal to zero. Then for the surplus product u_j of sector j it holds that[4],

$$u_j = X_j s_j - \bar{A} X_j s_j = (I - \bar{A}) X_j s_j \ . \qquad (15)$$

From (15) it is immediately seen that u_j is equal to the j-th column of $(I - \bar{A})$ multiplied by X_j, i.e. that

4. The surplus product u_j of sector j must not be confounded with that quantity U_j of commodity j produced by sector j, with which this commodity is represented in surplus product U of the whole production system.

$$u_j = X_j[-\bar{a}_{1j}, -\bar{a}_{2j}, \ldots, (1-\bar{a}_{jj}), \ldots, -\bar{a}_{nj}]^T \quad . \tag{15a}$$

Because of the irreducibitility of $\bar{A}$, some components of u_j must be negative. Nonetheless, profit pu_j and surplus value wu_j of this sector are always positive magnitudes.

<u>Proof</u>:

If wages are paid at the beginning of the production period, then from (9) we get for the price p^*s_j of commodity j produced by sector j

$$p^* s_j = (1+r^*)\,p^* \bar{A} s_j \ .$$

From this equation we get

$$p^* X_j s_j = (1+r^*)\,p^* \bar{A} X_j s_j \quad \Rightarrow$$

$$\frac{r^*}{1+r^*} p^* X_j s_j = r^* p^* \bar{A} X_j s_j =$$

$$= r^* p^* \bar{A} X_j s_j + p^* \bar{A} X_j s_j - p^* \bar{A} X_j s_j =$$

$$= (1+r^*)\,p^* \bar{A} X_j s_j - p^* \bar{A} X_j s_j =$$

$$= p^* X_j s_j - p^* \bar{A} X_j s_j =$$

$$= p^* (I - \bar{A}) X_j s_j =$$

$$= p^* u_j > 0 \tag{16}$$

The profit p^*u_j is obviously positive because it is equal to $\frac{r^*}{1+r^*} p^* X_j s_j$ and at the same time $r^* > 0$, $p^* > 0$ and $X_j s_j \geq 0$.

If wages are paid at the end of the production period, then from (9a) we get for the price $p^{**}s_j$ of the commodity j produced by sector j

$$p^{**}s_j = (1 + r^{**})p^{**}As_j + p^{**}d\ell s_j \,.$$

From this equation we get

$$p^{**}X_j s_j = (1 + r^{**})p^{**}AX_j s_j + p^{**}d\ell X_j s_j \quad \Rightarrow$$

$$p^{**}X_j s_j - p^{**}AX_j s_j - p^{**}d\ell X_j s_j = r^{**}AX_j s_j \quad \Rightarrow$$

$$p^{**}[I - A - d\ell]X_j s_j = r^{**}AX_j s_j \quad \Rightarrow$$

$$p^{**}(I-\bar{A})X_j s_j = r^{**}AX_j s_j \,.$$

$p^{**}(I-\bar{A})X_j s_j$ is obviously the profit $p^{**}u_j$ of sector j. Thus the profit pu_j of each sector is positive despite the fact that this sector's surplus product contains also some negative components.

It remains to be proven that in the given case the surplus value wu_j of sector j is also positive, despite the fact that this sector's surplus product contains some negative components.

For the surplus value wu_j of sector j we get, taking into account (13) and (15),

$$\begin{aligned} wu_j &= \ell(I-A)^{-1}u_j = \\ &= \ell(I-A)^{-1}(I-\bar{A})X_j s_j = \\ &= \ell(I-A)^{-1}(I-A-d\ell)X_j s_j = \\ &= \ell X_j s_j - \ell(I-A)^{-1}d\ell X_j s_j = \\ &= \ell X_j s_j - wd\ell X_j s_j = \\ &= \ell X_j s_j(1-wd). \end{aligned}$$

Since $\ell > 0$, $X_j s_j \geq 0$ and, as have already been shown, $wd<1$, it follows that

$$wu_j = \ell X_j s_j (1-wd) > 0. \tag{17}$$

The above are obviously also valid for production systems, which use a surplus-productive technique and are not, as in the above case, viable. They are, in other words, valid for every production system using a surplus-productive technique regardless of whether it is viable or not.

Consequently, it cannot be said that the profit of a production system, which, regardless of whether it is viable or not, uses a surplus-productive technique, or the profit of a sector of such a production system is positive because its surplus product is (semi)positive. On the contrary, it can be said that its profit is positive because, regardless of whether its surplus product is (semi)positive or contains some negative components, its surplus value is positive.

Nothing changes in the preceding argumentation, if we assume that the matrices A and $\bar{A}$ are reducible.

And also nothing changes in our critique of the neoricardian interpretation of profit, if we assume joint production.

From the above analysis we can therefore conclude that the neoricardian interpretation of profit, according to which profit is positive because surplus product is (semi)positive, can not be sustained, while on the contrary the marxian interpretation of profit, according to which profit is positive because surplus value is positive, can be sustained.

The neoricardian interpretation of profit does not hold, because, as we have shown, not even the necessary for its validity condition is satisfied, namely the condition that surplus product is always (semi)positive when profit is positive. This condition is only a necessary but not a sufficient one for the validity of the neoricardian interpretation of profit, because, even if it were satisfied, i.e. even if surplus product were always (semi)positive when profit is positive, an additional proof should be given of the existence of a causal relation between (semi)positive surplus product and positive profit.

For a similar reason we have concluded above that the marxian interpretation of profit can be sustained and not that it is necessarily correct. Our analysis has only shown that profit is always positive when surplus value is positive. But this is only a necessary, not a sufficient condition for the validity of the marxian interpretation of profit. The respective sufficient condition is obviously the existence of a causal relation between positive surplus value and positive profit. The existence of such a relation can be proven, but it has not been proven in the above analysis. That is why, although we consider the marxian interpretation of profit correct, we have only concluded that it is potentially correct.

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