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On the Position and the Slope of the w-r-Curve

by
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1. Introduction

The discussion between neoclassical and neoricardian economists, the so-called "Cambridge Controversy" (see Harcourt 1972), did not lead to the identification of the factors, which determine the relation between the nominal wage rate (w) and the profit rate (r), in the general case[1], i.e. the factors, which determine the slope of the w-r-curve as well as its position as determined by the maximum nominal wage rate and the maximum profit rate that is derived from this curve.

The aim of this paper is to explore these determining factors and highlight the implications of the analysis. In part 2 of this paper we deal with a given decomposable production system and the subsystems in which this is decomposed. In part 3 we treat the normalization of prices and we introduce the notions of the normalization commodity and the normalization subsystem. Normalization commodity we call the commodity or basket of commodities, the prices of which we set, in order to normalize the vector of relative prices, equal to a positive constant. And normalization subsystem we call the subsystem, which, using the same technique as the given production system, produces the chosen normalization commodity as its net product. In part 4 we derive the *w-r*-relation for the given production system. In part 5 we claim that this w-r-relation has always a form, from which it can be immediately inferred that it actually represents the *w-r*-relation of the respective normalization subsystem. This claim is proved in parts 6-8. In part 9 we prove that the slope and position of the *w-r*-relation depends, and we show how, on the price normalization and particularly on the chosen normalization commodity and the respective normalization subsystem. In part 10 we mention that, due to the fact that the *w-r*-relation is that of the respective normalization subsystem, in the general case,

a) the unambiguous ranking, comparison and choice of techniques with respect to their profitability are impossible, and
b) the phenomena of switching and reswitching of techniques appear and disappear according to the normalization used.

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1. By general case we mean here and in the subsequent analysis the set of all possible cases.

In part 11, finally, we answer definitely the so far unanswered question in what way does the price of a commodity or of a basket of commodities change, when the profit rate or the nominal wage rate changes.

2.The analytical framework: a decomposable production system

We are referring to the production system [A, ℓ, X], which produces the gross product X, X > 0, by using the productive technique [A, ℓ], where A, A ≥ 0, is the nxn matrix of the technological coefficients and ℓ, $\ell \geq 0$, is the 1xn row vector of the labour inputs per unit of commodity produces. For the matrix A we assume that it is nonsigular (i.e. rank (A)=n), decomposable, and has the following canonical form

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$$

This canonical form of A shows that, the given production system using the technique [A, ℓ], produces two kinds of commodities, namely basic commodities (called from now on *commodities of the 1st kind*) and non basic commodities, each one of which enters the production of all non-basic commodities, (called *commodities of the 2nd kind*). Let the system produce m commodities of the 1st kind and n-m commodities of the 2nd kind, where 1 ≤ m ≤ n-1. Consequently, A_{11} is an mxm matrix and A_{22} is an (n-m)x(n-m) matrix. Since rank(A)=n, we have rank(A_{11})=m and rank(A_{22})=n-m. Thus, the matrices, A_{11} and A_{22} are nonsingular. We furthermore assume that $A_{12} \geq 0$ and A_{22} is an indecomposable matrix.

Our assumption that the technique [A, ℓ] is productive implies that

$$\lambda_m^A < 1,$$

were λ_m^A is the maximum eigenvalue of A. We know that

$$\lambda_m^A = \max(\lambda_m^{A11}, \lambda_m^{A22}),$$

where λ_m^{A11} and λ_m^{A22} are the maximum eigenvalues of A_{11} and A_{22} respectively. We therefore have

$$\lambda_m^{A11} < 1$$

and

$$\lambda_m^{A22} < 1$$

Finally, we assume that

$$\lambda_m^{A11} \neq \lambda_m^{A22}$$

and that in the case, in which $\lambda_m^{A11} < \lambda_m^{A22}$, all the eigenvalues of A_{22}, except of course the maximum eigenvalue $\lambda_m^{A_{22}}$, are smaller than the maximum eigenvalue $\lambda_m^{A_{11}}$ of A_{11}.

We shall employ the following definitions:

1. r is the rate of profit of the given production system [A, ℓ, X] and of any of its subsystems.
2. R is the maximum rate of profit of the given production system [A, ℓ, X]. R is the rate of profit r which corresponds to zero nominal wage rate w.
3. $\bar{R}$ is the value of R which corresponds to positive or semipositive prices p of all the commodities produced by the system as its gross product. More specifically, $\bar{R}$ is the profit rate r obtained in the case, where the prices of all the commodities produced by the system as its *gross* product are *positive* or *semipositive* and the prices of the inputs of the system, other than the ones produced by the system itself, namely the price of labour power w, are equal to zero.

When the wages are paid post factum, the prices of the commodities satisfy the relation

$$p\,[I - (1 + r)\,A] = w\ell\,. \qquad (1)$$

By setting $w=0$ in (1), we get

$$p\,[I - (1 + r)\,A] = 0 \text{ for } w=0 \text{ and } r=R\,. \qquad (2)$$

From (2), it easily follows that

$$R = \frac{1-\lambda^A}{\lambda^A}$$

and

$$\bar{R} = \frac{1-\lambda_m^A}{\lambda_m^A}$$

where λ^A the eigenvalues of A.

One can easily see that $\bar{R}$ is the smallest positive value of R. $\bar{R}$ is also the value of the maximum rate of profit R of the given production system [A, ℓ, X] for which the relation $0 \le r \le \bar{R}$ quarantees *positive or semipositive* prices of all the commodities produced as its gross product (positive, when $0 \le r \le \bar{R}$, and positive or semipositive, when $r = \bar{R}$).

For the production subsystem I (production subsystem II), which produces as its net product commodities of the 1st kind only (commodities of the 2nd kind only), we define:

1. R_I (R_{II}) as the maximum rate of profit of the subsystem I (of the subsystem II) and

2. $\bar{R}_I$ $(\bar{R}_{II})$ as the value of R_I (of R_{II}), which corresponds to *positive* prices p_I (to *positive* prices p_{II}) of all the commodities produced by the subsystem I (by the subsystem II) as its *net* product. More precisely, $\bar{R}_I$ $(\bar{R}_{II})$ is the value of the profit rate r of the subsystem I (of the subsystem II) obtained, when the prices p_I (p_{II}) of all the commodities produced by the susystem I (by the subsystem II) as its *net* product are *positive*, and the prices of the inputs of the subsystem I (of the subsystem II), other than the ones produced by the subsystem I (by the subsystem II) as its *net* product are equal to zero.

Combining (2) with the decomposability of A we obtain

$$p_I [I - (1 + R) A_{11}] = 0 \quad \text{(a)}$$

and

$$p_{II} [I - (1 + R) A_{22}] = p_I (1 + R) A_{12} . \quad \text{(b)}$$

From (a) and (b) one can easily derive

$$R_I = \frac{1-\lambda^{A_{11}}}{\lambda^{A_{11}}},$$

$$\bar{R}_I = \frac{1-\lambda_m^{A_{11}}}{\lambda_m^{A_{11}}},$$

$$R_{II} = \frac{1-\lambda^{A_{22}}}{\lambda^{A_{22}}},$$

and

$$\bar{R}_{II} = \frac{1-\lambda_m^{A_{22}}}{\lambda_m^{A_{22}}}$$

where $\lambda^{A_{11}}$ and $\lambda^{A_{22}}$ are the eigenvalues of A_{11} and A_{22} respectively.

One can also see that $\bar{R}_I$ is the smallest positive value of R_I, i.e. $R_I \geq \bar{R}_I$ and $\bar{R}_{II}$ is the smallest positive value of R_{II}. $\bar{R}_I$ $(\bar{R}_{II})$ is also the value of the maximum rate of profit R_I (R_{II}) of the production subsystem I (of the production subsystem II), for which the relation $0 \leq r \leq \bar{R}_I$ (the relation $0 \leq r \leq \bar{R}_{II}$) guarantees *positive* prices p_I (p_{II}) of all the commodities produced by the subsystem I (by the subsystem II) as its *net* product.

What has been said above about the given production system [A, ℓ, X] holds also for every subsystem [A, ℓ, V] of the given production system, which uses the same techniques [A, ℓ] as the one used by the given production system and produces, as this one, all n commodities, the production of which is made possible by the use of technique [A, ℓ], only in different quantities, i.e. V > 0 and V ≠ X. If r is the profit rate, R_V is the maximum profit rate of the subsystem [A, ℓ, V] and $\bar{R}_V$ is the smallest positive value of R_V, then we have R_V = R and $\bar{R}_V = \bar{R}$. Therefore, for the prices *p* of all the commodities which the subsystem [A, ℓ, V] produces as its *gross* product, it holds what has been said about the prices *p* of the same commodities produced by the given production system [A, ℓ, X] as its *gross* product.

3. Normalization of prices

Equation (2) has a solution, other than the trivial one p = 0, when rank([I - (1 + R) A]) < n, where n is the order of A and hence of [I - (1 + R) A].

One can show (see Stamatis 1991, vol. 1, p. 67ff.) that, when rank(A) = n, then

$$\mathrm{rank}([I - (1 + R)\,A]) = n - 1\,. \tag{3}$$

Since (3) holds, equation (2) has a solution other than the trivial one. This solution of (2) gives n values for R, to each of which corresponds a price vector p uniquely determined up to multiplication by a scalar.

Due to (3), there exists the inverse of [I - (1 + R) A] for each r, r ≥ 0 and r ≠ R, and correspondingly for each w, $0 < w \leq w_{max}$, where $w_{max} = w_{(r=0)}$. So from (1) we obtain

$$p = w\ell[I - (1 + r)\,A]^{-1},\ \text{for}\ 0 < w \leq w_{max}\ \text{or}\ 0 \leq r\ \text{and}\ r \neq R\,. \tag{4}$$

It is obvious that (4) determines the price vector p uniquely and up to a scalar for an exogenously given r, where w it does not determine it for an exogenously given w:

We therefore conclude that (1) or (2) and (4) do not fully determine the price vector p for an exogenously given r or w. Either they don't determine it or they determine it only up to a scalar. In order to obtain the *w-r*-relation from (1) or from (2) and (4), the price vector p must be fully determined either for a given w or for a given r.

So, in order to obtain the *w-r*-relation from (1) or from (2) and (4), we must arbitrarily determine the above scalar. This is done through the arbitrary determination of the price of one commodity or of a basket of commodities.

We call this arbitrary determination of the price of a commodity, or of a basket of commodities, price normalization. The price normalization is accomplished by equating the price of a commodity, or of a basket of commodities, to a constant positive quantity of a homogeneous extensive thing. We call this equation, this commodity, or basket of commodities, and this homogeneous extensive thing normalization equation, normalization commodity and fictitious money respectively. None of the commodities, which the given production system produces, except the normalization commodity, can function as fictitious money. Otherwise the system of equations, which determines prices, will be overdetermined.

Let y, $y \geq 0$, be the normalization commodity. If we set the price of this normalization commodity equal to b units of a homogeneous extensive thing B, which is not produced by the given production system, then the price normalization equation is

$$py = b, \text{ where } y \geq 0, \tag{5}$$

where b is a positive constant quantily of B. B is the fictitious money, in terms of which the absolute prices, as well as the other nominal quantities, are expressed.

We call normalization subsystem the production subsystem which produces the chosen normalization commodity y as its net product, using the same technique as that of the given production system. The gross product x of the above normalization subsystem is obviously

$$x = (I - A)^{-1} y \geq 0. \tag{6}$$

4. The *w-r*-relation

Postmultiplying equation (1) by y and taking (5) into account, we obtain

$$w = \frac{b}{\ell\left[I-(1+r)A\right]^{-1}y}, \text{ for } 0 < w \leq w_{max} \text{ or } r \geq 0 \text{ and } r \neq R, \tag{7}$$

and

$$R = \frac{b - pAy}{pAy} = \frac{p\left[I-A\right]y}{pAy}, \text{ for } r = R. \tag{8}$$

Equations (7) and (8) constitute the *w-r*-relation.

Eliminating from vectors x, y, ℓ and p those components, which are equal to zero in the vector x, i.e. those components, which are related to commodities produced by the given production system, but not by the normalization subsystem, we obtain the vectors x^*, y^*, ℓ^* and p^*, respectively. Accordingly, eliminating all columns and the respective rows of A, which are related to the above commodities, we obtain the matrix A^*.

A^* is that part of A (which may coincide with A), which constitutes the matrix of technological coefficients of the normalization subsystem. ℓ^* is that part of ℓ (which may coincide with ℓ), representing the labour inputs per unit of commodities produced by the normalization subsystem and of these commodities only. The technique $[A^*, \ell^*]$ is that part of technique $[A, \ell]$ (which may coincide with $[A, \ell]$), that is used by the normalization subsystem. Finally, p^* is that part of price vector p (which may coincide with p), constituting the price vector of commodities produced by the normalization subsystem and of these commodities only.

Depending on the kind of normalization commodity, we distinguish two types of normalizations:

Normalization of the 1st type: The normalization commodity consists only of commodities of the 1st kind. In this case the normalization subsystem produces as its gross product commodities of the 1st kind only.

Normalization of the 2nd type: The normalization commodity consist of commodities of the 2nd kind or of commodities both of the 1st kind and of the 2nd kind. In this case the normalization subsystem obviously produces as its *gross* product all the commodities of the given production system. In this case the normalization subsystem is a subsystem of the type of the above mentioned subsystem $[A, \ell, V]$.

From the above it follows that

$$A^* = A_{11}$$

for normalizations of the 1st type and

$$A^* = A$$

for normalizations of the 2nd type.

Let R^* be the maximum rate of profit of the normalization subsystem and $\bar{R}^*$ be the value of R^*, which guarantees positive or non-negative prices of the commodities produced by the normalization subsystem as its gross product for each r, $0 \leq r \leq \bar{R}^*$, (positive for each r, $0 \leq r \leq \bar{R}^*$, and positive or non-negative for $r = \bar{R}^*$). Then it holds:

$$R^* = R_I$$

and

$$\bar{R}^* = \bar{R}_I$$

for normalizations of the 1st type and

$R^* = R$

and

$\bar{R}^*\left[=\min(\bar{R}_I,\bar{R}_{II})\right] = \bar{R}\left[=\min(\bar{R}_I,\bar{R}_{II})\right]$

for normalizations of the 2nd type. Depending on the type of price normalization, we have either

$\bar{R}^* = \bar{R}_I$

or

$\bar{R}^* = \bar{R}_{II}$.

Moreover, for a normalization of the 1st type we have

$\bar{R}^*(=\bar{R}_I) \geq \bar{R}\left[=\min(\bar{R}_I,\bar{R}_{II})\right]$.

More specifically, when

$\bar{R}_I > \bar{R}_{II}$,

then we have

$\bar{R}^*(=\bar{R}_I) > \bar{R}\left[=\min(\bar{R}_I,\bar{R}_{II}) = \bar{R}_{II}\right]$

and, when

$\bar{R}_I < \bar{R}_{II}$,

then we have

$\bar{R}^*(=\bar{R}_I) = \bar{R}\left[=\min(\bar{R}_I,\bar{R}_{II}) = \bar{R}_{II}\right]$.

Thus, according to a normalization of the 1st type, it is possible to have

$\bar{R}^*(=\bar{R}_I) \neq \bar{R}\left[=\min(\bar{R}_I,\bar{R}_{II})\right]$.

On the contrary, for a normalization of the 2nd type, it always holds that

$\bar{R}^*\left[=\min(\bar{R}_I,\bar{R}_{II})\right] = \bar{R}\left[=\min(\bar{R}_I,\bar{R}_{II})\right]$.

Consequently, for a normalization of the 2nd type, when

$\bar{R}_I > \bar{R}_{II}$,

then we have

$\bar{R}^*(=\bar{R}_{II}) = \bar{R}(=\bar{R}_{II})$

and, when

$\bar{R}_I < \bar{R}_{II}$,

then we have

$\bar{R}^*(=\bar{R}_I) = \bar{R}(=\bar{R}_I)$

According to the above analysis, equations (1), (2), (3), (4), (5), (6), (7), and (8) are substituted by the following

$$p^*[I^*-(1+r)A^*]=w\ell^* \tag{1a}$$

$$p^*[I^*-(1+R^*)A^*]=0, \text{ for } w=0 \text{ or } r=R^*, \tag{2a}$$

$$\text{rank}([I^*-(1+R^*)A^*])=k-1, \tag{3a}$$

where k, $1<k\leq n$, the order of A^* and consequently of $[I^*-(1+R^*)A^*]$,

$$p^*=w\ell^*[I^*-(1+r)A^*], \text{ for } 0<w\leq w_{max} \text{ or } 0\leq r \text{ and } r\neq R^* \tag{4a}$$

$$p^*y^*=b, \text{ where } y^*\geq 0, \tag{5a}$$

$$w=\frac{b}{\ell^*[I^*-(1+r)A^*]^{-1}y^*}, \text{ for } 0<w\leq w_{max} \text{ or } 0\leq r \text{ and } r\neq R^*, \tag{7a}$$

and

$$R^*=\frac{b-p^*A^*y^*}{p^*A^*y^*}=\frac{p^*[I^*-A^*]y^*}{p^*A^*y^*}, \text{ for } w=0 \text{ or } r=R^*. \tag{8a}$$

5. Claims to be proven

We will prove that the *w-r*-relation given by (7a) and (8a) is the *w-r*-relation of the normalization subsystem, which also holds as the *w-r*-relation of the given production system as well as the *w-r*-relation of every other production system, which uses the same technique as that of the given production system, but produces a different (gross and net) product.

We will show that for any *w-r*-relation we always have

$$w=v_n-\pi_n-v_n-\frac{\pi_n}{k_n}k_n=v_n-k_n r, \tag{9}$$

where

v_n is the constant average labour productivity in price terms and consequently the maximum wage rate of the normalization subsystem,

π_n is the average profit per unit of labour in the normalization subsystem,

k_n is the average capital intensity of the normalization subsystem in price terms,

w is the uniform nominal wage rate of the normalization subsystem, and
r is the uniform profit rate of the normalization subsystem.

w and r represent the uniform nominal wage rate and the uniform profit rate not only of the normalization subsystem but also of the given production system. This is a consequence

(a) of the implicit axiom of the existence of a uniform nominal wage rate and of a uniform profit rate for all quantities of the same commodity as well as for all commodities and

(b) of the fact that, to the extent that they produce the same commodities, the given production system and the normalization subsystem use the same *linear* technique.

For the same reasons the *w-r*-relation of the normalization subsystem holds also as the *w-r*-relation of the given production system. That the *w-r*-relation of the normalization subsystem holds also as the *w-r*-relation of the given production system can be seen from the fact that the w-r-relation, as represented by (7) and (8) or by (7a) and (8a), does not depend on the gross product X or on the net product Y of the given production system, but depends only on the gross product x or on the net product y of the normalization subsystem.

From equation (9) we obtain

$$\frac{dw}{dr} = \frac{dv_n}{dr} - \frac{dk_n}{dr} r - k_n. \tag{10}$$

Because

(a) the basket of commodities, the price of which v_n represents, consists of net product of the normalization subsystem, i.e. of normalization commodity, and consequently is a constant fraction or a constant multiple of the normalization commodity (a constant fraction, if the normalization subsystem uses more than one unit of labour, and a constant multiple, if the normalization subsystem uses less than one unit of labour), and

(b) the size of this basket of commodities of unchanged composition, i.e. the size of the average net product of a labour unit used in the normalization subsystem, if given by the conditions of production in the normalization subsystem and therefore constant and independent of the rate of profit,

we have

$$\frac{dv_n}{dr} = 0. \tag{11}$$

The price v_n of this basket of commodities remains unchanged when the rate of profit varies, due to the same reasons, for which the price py or p^*y^* of the normalization commodity y or y^* respectively remains unchanged when the rate of profit varies.

From (10) and (11) we obtain the slope of the *w-r*-relation as

$$\frac{dw}{dr}=\frac{dk_n}{dr}r-k_n. \qquad (12)$$

From (12) we conclude that for a given r the slope of the *w-r*-relation depends exclusively on the average capital untensity of the normalization subsystem in price terms k_n and on its rate of change with respect to the rate of profit dk_n / dr.

When the average capital intensity of the normalization subsystem in price terms k_n does not change with the rate of profit, i.e. when k_n=constant, then $dk_n/dr=0$ and consequently

$$\frac{dw}{dr}=-k_n. \qquad (13)$$

In this case the *w-r*-relation is linear and its slope is equal to $-k_n$.

From (9) we obtain

$$w_{max} = w_{(r=0)} = v_n \qquad (14)$$

and

$$R^* = r_{(w=0)} = v_n / k_n. \qquad (15)$$

(14) means that the maximum nominal rate w_{max}, which is obtained from the *w-r*-relation, is always equal to the constant average labour productivity in the normalization subsystem in price terms and, because the maximum nominal wage rate of every production system or subsystem is nothing else but the average labour productivity of this production system or subsystem in price terms when r=0, it is also equal to the maximum nominal wage rate of the normalization subsystem.

The ratio v_n / k_n represents the maximum rate of profit of the normalization subsystem. Consequently, (15) means that the maximum rate of profit R^*, which is obtained from the *w-r*-relation, is always equal to the maximum rate of profit of the normalization subsystem.

6. The position and the slope of the *w-r*-curve

We will prove that each *w-r*-relation is of the form given by (9). In order to prove that, it is sufficient to prove that the *w-r*-relation, described by equations (7a) and (8a), is of the form given by (9).

To this aim, we derive the average labour productivity in price terms and the average capital intensity in price terms of the normalization subsystem. We obtain for the former

$$v_n=\frac{b}{\ell^*(I^*-A^*)y^*} \qquad (16)$$

and for the latter

$$k_n = \frac{p^* A^* (I^* - A^*)^{-1} y^*}{\ell^* (I^* - A^*)^{-1} y^*} \tag{17}$$

Substituting (16) and (17) into (9), we obtain

$$w = \frac{b}{\ell^* (I^* - A^*)^{-1} y^*} - r \frac{p^* A^* (I^* - A^*)^{-1} y^*}{\ell^* (I^* - A^*)^{-1} y^*}. \tag{18}$$

Combining (4a) with (18) for w, $0 < w \leq w_{max}$ or r, $0 \leq r < R^*$, we obtain

$$w = \frac{b}{\ell^* (I^* - A^*)^{-1} y^* + r \ell^* [I^* - (1+r)A^*]^{-1} (I^* - A^*)^{-1} y^*}. \tag{19}$$

(19) is identical to (7a), provided that it holds

$$\ell^* (I^* - A^*)^{-1} y^* + r \ell^* [I^* - (1+r)A^*]^{-1} A^* (I^* - A^*)^{-1} y^* = \ell^* [I^* - (1+r)A^*]^{-1} y^*. \tag{20}$$

This last equation holds because

$$\left\{ [I^* - (1+r)A^*]^{-1} A^* r + I^* \right\} (I^* - A^*)^{-1} y^* = [I^* - (1+r)A^*]^{-1} y^* \Leftrightarrow$$

$$\left\{ A^* r + [I^* - (1+r)A^*] \right\} (I^* - A^*)^{-1} y^* = y^* \Leftrightarrow$$

$$\left\{ A^* r + I^* - A^* - A^* r \right\} (I^* - A^*)^{-1} y^* = y^* \Leftrightarrow$$

$$(I^* - A^*)(I^* - A^*)^{-1} y^* = y^* \Leftrightarrow$$

$$y^* = y^*.$$

So, (19) is identical to (7a).
Combining (6a) with (18) for w=0 or $r = R^*$ we obtain

$$R^* = \frac{b}{p^* A^* x^*} = \frac{p^* (I^* - A^*) x^*}{p^* A^* x^*} \tag{21}$$

Claim: Equation (21) is identical to Equation (8a).
Proof: From (8a) we obtain

$$p^* [I^* - (1 + R^*) A^*] y^* = 0. \tag{22}$$

From (21) we obtain

$$p^* [I^* - (1 + R^*) A^*] x^* = 0. \tag{23}$$

Equation (21) is identical to (8a), when

$$p^*[I^*-(1+R^*)A^*]y^* = p^*[I^*-(1+R^*)A^*]x^*. \qquad (24)$$

This last equation holds because

$$p^*[I^*-(1+R^*)A^*] = 0. \qquad (2a)$$

Consequently we have

$$R^* = \frac{p^*(I^*-A^*)x^*}{p^*A^*x^*} = \frac{p^*(I^*-A^*)y^*}{p^*A^*y^*} \qquad (25)$$

From (21) and (25) one can immediately see that the maximum rate of profit R^* which is obtained from the *w-r*-relation, is equal to the maximum rate of profit of the normalization subsystem.

We have therefore proved that the w-r-relation given by (7a) and (8a), and consequently every *w-r*-relation, is of the form described by (9). At the same time we have proved that (14) and (15) generally hold. We also have proved *indirectly* that (12) holds in general. Nevertheless we wish to prove, in a more direct way, that (12) holds generally, by determing the slope of the *w-r*-relation described by (7a) and by showing that for that slope (12) holds.

7. The slope of the *w-r*-curve

Taking (1a) and (5a) into consideration, from (7a) we obtain

$$\frac{dw}{dr} = -\frac{p^*A^*[I^*-(1+r)A^*]^{-1}y^*}{\ell^*[I^*-(1+r)A^*]^{-1}y^*}, \text{ for r, } 0 \le r < R^*. \qquad (26)$$

We will prove that (12) is identical to (26). To this aim, we derive dk_n / dr with the help of (17), (1a) and (5a) and we substitute it together with k_n of (17) in equation (12).

Following some manipulations we obtain

$$\frac{dw}{dr} = -\frac{p^*A^*\left\{[I^*-(1+r)A^*]^{-1}A^*(I^*-A^*)^{-1}y^*r + (I^*-A^*)^{-1}y^*\right\}}{\ell^*\left\{[I^*-(1+r)A^*]^{-1}A^*(I^*-A^*)^{-1}y^*r(I^*-A^*)^{-1}y^*\right\}}.$$

Combining the above equation with (20), we obtain (26). Thus we have proven that (12) holds in general.

8. The maximum rate of profit which is obtained from the *w-r*-relation

The *w-r*-relation which is given by (7a) and (8a), when it is not linear, is not a one to one relation[2]. Specifically, to each value of r corresponds only one value of w, but to each value of w correspond in general either one or more than one values of r.

Let us examine this point more closely.

If prices have been normalized with a normalization of the 1st type, then $A^*=A_{11}$, and the relations (4a), (2a), (3a), (5a), (7a) and (8a) take the form

$$p_I^*=w\ell_I^*[I-(1+r)A_{11}]^{-1}, \text{ for } 0<w\leq w_{max} \text{ or } 0\leq r<R_I^*, \tag{4b}$$

$$p_I^*[I-(1+R_I)A_{11}]=0, \text{ for } w=0 \text{ or } r=R_I, \tag{2b}$$

$$\text{rank}([I-(1+R_I)A_{11}])=m-1, \tag{3b}$$

where m is the order of A_{11},

$$p_I^*y_I^*=b, \text{ where } y_I^*\geq 0, \tag{5b}$$

$$w=\frac{b}{\ell^*[I-(1+r)A_{11}]^{-1}y_I^*}, \text{ for } 0<w\leq w_{max} \text{ or } 0\leq r \text{ and } r\neq R_I, \tag{7b}$$

and

$$p_I^*=[I-(1+R_I)A_{11}]y_I^*=0, \text{ for } w=0 \text{ or } r=R_I, \tag{8b}$$

where $p_I^*(=p_I)$ is the price vector of the commodities of the 1st kind, $\ell_I^*(=\ell_I)$ is the vector of labour inputs per unit of produced commodities of the 1st kind, and y_I^* is the vector of the normalization commodity, which consists only of commodities of the 1st kind.

From (7b) we can see by inspection that to each value of w, $0<w\leq W_{max}$, correspond in general up to k (=m) values of r.

From (8b), or even clearer (2b), from which we have obtained (8b), we can see by inspection that to the value of w, w=0, correspond in general up to k (=m) values of r ($=R_I$).

2. The *w-r*-relation is an one to one relation, when it is (either for every normalisation or for a suitable normalisation) linear.

In the special case, in which the normalization of the 1st type is Sraffa's normalization (see Sraffa 1960, §23ff) or, even more generally, Miyao's normalization (see Miyao 1977 and Stamatis/Dimakis 1981), w is a first degree function in r, both in (7a) and (8a). In particular, to the value of w, w=0, corresponds according to (8b) one value of r, namely the value $r=\bar{R}_I$, and to each value of w, $0<w\leq w_{max}$, corresponds according to (7b) one value of $r, 0\leq r<\bar{R}_I$.

If the prices are normalized with a normalization of the 2nd type

$$p^* y^* = p_I^* y_I^* + p_{II}^* y_{II}^* = b,\ y_I^* \geq 0,\ y_{II}^* \geq 0 \tag{5c}$$

we distinguish two subcases: Either

$\bar{R}_I < \bar{R}_{II}$ (subcase (i))

or

$\bar{R}_I > \bar{R}_{II}$ (subcase (ii)).

We can write (2a), from which we have obtained (8a), as follows:

$$p_I^*[I-(1+R^*)A_{11}]=0 \tag{2c}$$

and

$$p_{II}^*[I-(1+R^*)A_{22}]=p_I^*(1+R^*)A_{12}. \tag{2cc}$$

In subcase (i), (2c) gives $R^*=R_I$ and $p_I^* \neq 0$, and (2cc) gives, for $R^*=R_I$ and $p_{II}^* \neq 0$. So in subcase (i) (8a) gives, for w=0, k(=m) values for $r=R^*(=R_I)$, and gives for each value of w, $0<w<w_{max}$, k(=m) values for r.

In subcase (ii), (2c) gives $p_I^* = 0$ for each R^*. For $p_I^* = 0$, (2cc) takes the form

$$p_{II}^*[I-(1+R^*)A_{22}]=0$$

and gives for $p_{II}^* \neq 0$, $R^*=R_{II}$. So in subcase (ii), (8a) gives for w=0, k(=m) values for $r(=R^* =R_{II})$ and (7a) gives for each value or w, $0<w\leq w_{max}$, k(=n-m) values for r.

If in subcase (ii) prices are normalized according to Vassilakis's normalization (see Vassilakis 1983, Stamatis 1991, vol. 1, p. 259ff), or even more generally, according to Mariolis/Voujouklakis's normalization (see Mariolis/Voujouklakis 1992), then for each value or w, $0\leq w\leq w_{max}$, (8a) and (7a) give only one value for r.

So, if prices are normalized with a normalization of the 2nd type, then, according to (7a) and (8a), to each value of w, $0\leq w\leq w_{max}$, correspond to subcase (i) up to m and in subcase (ii) up to n-m values for r. And only if prices are normalized according to Vassilakis or, even more generally, according to Mariolis/Voujouklakis, corresponds, in subcase (ii), to each value of w, $0\leq w\leq w_{max}$, only one value for r.

The *w-r*-relation is not in the general case an one to one relation. Usually one simply presupposes that the w-r-relation is a one to one relation even in the case, in which it is not linear. In fact there exists a way to secure that the *w-r*-relation is a one to one relation. This can be done by introducing an upper bound $\bar{R}^*$ for the rate of profit. If we restrict the rate of profit to vary in the interval $[0,\bar{R}^*]$, i.e.

$$0 \leq r \leq \bar{R}^* ,$$

then the *w-r*-relation is always a one to one relation. The lower bound zero of r has a profound economic interpretation. The upper bound $\bar{R}^*$ also has an economic interpretation. Its meaning consists in the fact that for $0 \leq r \leq \bar{R}^*$, the prices p^* of the commodities produced by the normalization subsystem are positive or non-negative[3]. For, as we know, p^* is positive or non-negative for each r, $r = \bar{R}^*$, and positive for each $0 \leq r \leq \bar{R}^*$.

We know that, when the *w-r*-relation is not linear, to each w, $0 \leq w \leq w_{max}$, corresponds more than one values of r. Only to the smallest positive (when $0 \leq w \leq w_{max}$) or non-negative (when $w = w_{max}$) of these values of r corresponds a positive or non-negative p^*, whereas to the rest of them, which are larger than $\bar{R}^*$, for w, $0 \leq w \leq w_{max}$, correspond p^*'s with negative components (see Mariolis 1992). It is impossible for one or these values of r, to which correspond p^*'s with negative components, to be equal or less than $\bar{R}^*$, because

$$0 \leq r \leq \bar{R}^* \Leftrightarrow p^* > 0 \text{ or } p^* \geq 0.$$

3. At the same time, it is possible to get negative prices for the commodities produced only by the production system (but not also by the normalisation subsystem). For example, when $\bar{R}_{II} < \bar{R}_I$ and the prices are normalised using a normalisation of the 1st type, the prices $p_I^* = p_I$ of the commodities produced by the normalisation subsystem are positive for each r, $0 \leq r \leq \bar{R}^* = \bar{R}_I$, while the prices p_{II} of the commodities produced only by the given production system are positive for each r, $0 \leq r \leq \bar{R}_{II}$, indeterminate for $r = \bar{R}_{II}$ and either negative or of ambignous sign for each r, $\bar{R}_{II} < r \leq \bar{R}_I$. One can obtain positivity or non-negativity of the prices of all commodities by including in the normalisation commodity at least one commodity, which is produced as net product by the subsystem with the smallest $\bar{R}_j$, $j = I, II$.

Therefore for r, $0 \leq r \leq \bar{R}^*$, the *w-r*-relation is always a one to one relation.

The *w-r*-relation *as an one to one correspondence* is defined by (7a), (8a) and $R = \bar{R}^*$. The maximum rate of profit of the *w-r*-relation as *an one to one correspondence* is $\bar{R}^*$, i.e. the smallest positive value of the maximum rate of profit of the normalization subsystem R^*.

9. The effects of changes in the normalization on the *w-r*-relation

We showed that the maximum rate of profit $\bar{R}^*$, which is obtained from the *w-r*-relation as *an one to one correspondence,* changes with the type of normalization used: For a normalization of the 1st type, $\bar{R}^*$ is always equal to $\bar{R}_I$, whereas for a normalization of the 2nd type it is equal to $\bar{R}_I$, when $\bar{R}_I < \bar{R}_{II}$, and equal to $\bar{R}_{II}$, when $\bar{R}_I > \bar{R}_{II}$. So, when $\bar{R}_I > \bar{R}_{II}$ and we use a normalization of the 1st type, we have $\bar{R}^* = \bar{R}_I$, while, when $\bar{R}_I > \bar{R}_{II}$ and we use a normalization of the 2nd type, we have $\bar{R}^* = \bar{R}_{II}$. The maximum rate of profit $\bar{R}^*$, which is obtained from the *w-r*-relation as *an one to one correspondence,* changes with the normalization, when the normalization commodity is changed in such a way as to include (not include) commodities of the 2nd kind, which it did not include (did include) up to now.

However, the *w-r*-relation as *an one to one correspondence* does not change with the kind of fictitious money. When the normalization commodity does not change, but the quantity of the fictitious money b to which the price of the normalization commodity is equated increases (decreases), then, as it can be seen from (7a) and the relation $R^* = R^*$, the *w-r*-curve as *an one to one correspondence* rotates around the point $r = \bar{R}^*$ to the right (to the left), and as a consequence the maximum nominal wage rate w_{max} and the absolute value of the slope of the *w-r*-relation -dw/dr increase (decrease).

Finally, when the normalization commodity changes, ceteris paribus, then, irrespective of whether it changes in a way that implies or does not imply a change in $\bar{R}^*$, w_{max} and -dw/dr change, because the average labour productivity in the normalization subsystem in price terms and the capital intensity in the normalization subsystem in price terms change.

10. The implications of the fact that the *w-r*-relation is that of the respective normalization subsystem

We have seen that the *w-r*-relation is that of the normalization subsystem. This explains the fact that it changes with every change in the normalization commodity.

That the *w-r*-relation is the *w-r*-relation of the normalization subsystem implies that in the general case the ranking of given decomposable techniques with respect to their profitability changes with the normalization, and therefore an unambiguous ranking of these techniques is impossible. As an implication of this the comparison of decomposable techniques with respect to their profitability and the choice of the most profitable among them are impossible in the general case.

The usual ranking and comparison of techniques with respect to their profitability and the choice of the most profitable among them is actually a ranking and comparison of normalization subsystems with respect to their profitability and the choice of the most profitable among them, namely of the normalization subsystems, which use the given techniques in order to produce the chosen normalization commodity as their net product. That is why, in the general case, this ranking, this comparison and this choice of techniques change with the normalization, when the normalization commodity changes with the normalization (see Stamatis 1990 and Stamatis 1991, vol. 1, p. 305ff).

Furthermore, when the given techniques are decomposable, the phenomena of switching and of reswitching of techniques appear and disappear with the normalization in the general case (see Stamatis 1990).

Due to this fact, it must be re-examined to what extent the neoricardian critique of the neoclassical aggregate production function based on the phenomenon of reswitching of techniques (see Samuelson 1962 and Garegnani 1970) remains intact.

In production systems with decomposable techniques, the relative prices change in the general case with a change in the normalization (see Stamatis 1991, vol. 1, p. 159ff and p. 302ff) -a fact, which perhaps is not without consequence for the general equilibrium theory.

Finally, in production systems with decomposable techniques in the general case the general rate of profit does not depend only on the conditions of production of the basic commodities, but on the conditions of production of the non-basic commodities as well (see Stamatis 1991, vol. 1, p. 298ff and vol. 2, p. 189ff).

11. Changes in prices due to changes in the profit rate or in the nominal wage rate

One further significant result of the fact that the *w-r*-relation is always the *w-r*-relation of the respective normalization subsystem is the following: It allows us to answer definitively the so far unanswered question of how the price pz of a commodity or basket of commodities z, $z \geq 0$, changes, when the rate of profit r or the nominal wage rate w changes.

For the change in the price pz of commodity z we obtain from (1) and (5)

$$\frac{d(pz)}{dw}\frac{dw}{dr} = \frac{d(pz)}{dr} = \left(pA + \frac{dw}{dr}\ell\right)\left[I-(1+r)A\right]^{-1} z. \qquad (27)$$

From the above relation, taking into account the equations (12), (20) and

$$k_z = \frac{pA(I-A)^{-1}z}{\ell(I-A)^{-1}z},$$

where k_z is the capital intensity of z in price terms, we obtain

$$\frac{d(pz)}{dr} = \ell(I-A)^{-1}z\left[\left(\frac{dk_z}{dr}r + k_z\right) - \left(\frac{dk_n}{dr}r + k_n\right)\right].$$

From the above relation we can easily conclude that

$$\frac{d(pz)}{dr} \gtreqless 0 \Leftrightarrow \frac{dk_n}{dr}r + k_n \gtreqless \frac{dk_z}{dr}r + k_z \qquad (28)$$

and

$$\frac{d(pz)}{dr}\frac{dr}{dw} \gtreqless 0 \Leftrightarrow \frac{dk_n}{dr}r + k_n \gtreqless \frac{dk_z}{dr}r + k_z \qquad (29)$$

The following obviously hold

$$\frac{dk_n}{dr}r + k_n = \frac{d(k_n r)}{dr} = \frac{d\pi_n}{dr}$$

and

$$\frac{dk_z}{dr}r + k_z = \frac{d(k_z r)}{dr} = \frac{d\pi_z}{dr},$$

where $k_n r(=\pi_n)$ is the average profit per unit of direct labour in the normalization subsystem, $k_z r(=\pi_z)$ is the average profit per unit of direct labour in the subsystem which produces commodity z as its net product, $(dk_n/dr)r+k_n$ is the ratio of the marginal change of the average profit per unit of direct labour in the normalization subsystem to the marginal change of r and $(dk_z/dr)r+k_z$ is the ratio of the marginal change of the average profit per unit of direct labour in the subsystem producing z as its net product to the marginal change of r.

Thus, equations (28) and (29) allow us to infer how the price of a commodity changes, when r or w changes (see also Stamatis 1991, vol. 1, p. 292ff).

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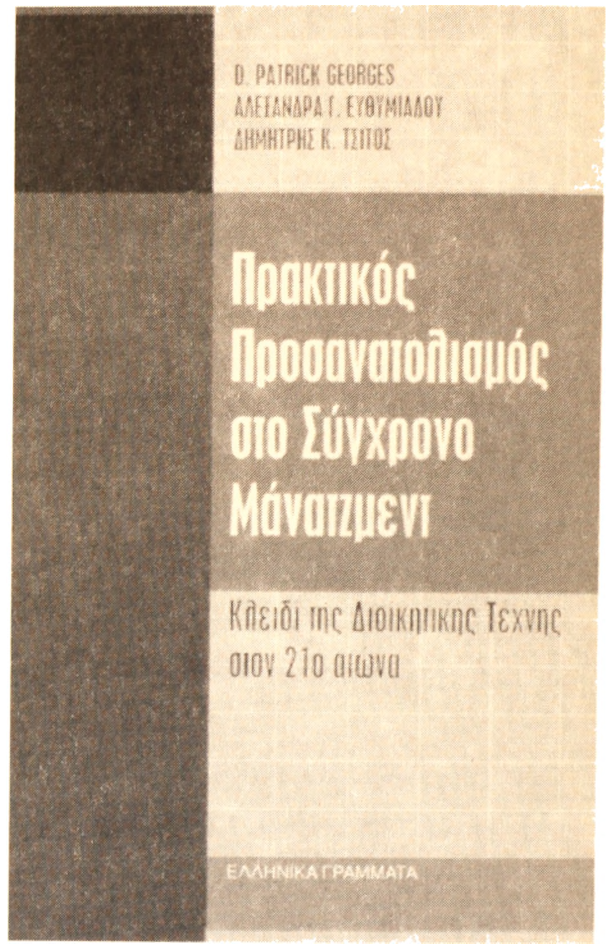
κυκλοφορεί

ΠΡΑΚΤΙΚΟΣ ΠΡΟΣΑΝΑΤΟΛΙΣΜΟΣ ΣΤΟ ΣΥΓΧΡΟΝΟ ΜΑΝΑΤΖΜΕΝΤ



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