POLITICAL ECONOMY - 2 - SPRING 1998 - PP. 147 - 167

The Impossibility of a Comparison of Techniques and of the Ascertainment of a Reswitching Phenomenon

A Reply to Erreygers and Kurz/Gehrke

by
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Two critiques, one by Erreygers (Erreygers, 1994) and one by Kurz/Gehrke (Kurz/Gehrke, 1994) were published in *Jahrbücher für Nationalökonomie und Statistik* concerning an article of mine published in the same journal (Stamatis, 1993).

The following is my response to these critiques.

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I will provide a brief reminder of the subject. In my article I did the following:

a) I showed that if one orders given decomposable techniques of single production according to the criterion of the w-r relation, i.e. for a given and uniform for all techniques rate of profit (r) with respect to the level of nominal wage rate (w), then, in the general case, this ordering varies with price normalization and, specifically, with the normalization commodity.

b) I showed that, as a result of the above, phenomena of switching and reswitching of techniques which appear for a certain normalization disappear for another normalization and conversely, phenomena of switching and reswitching of techniques which do not appear for a certain normalization, appear for another normalization.

c) I explained these apparent paradoxes by showing that the above ordering of given techniques with respect to their profitability is actually not an ordering of the techniques themselves, but of the corresponding normalization subsystems with respect to their profitability, i.e. of the subsystems each of which produces, using one of the given techniques, as its net product the normalization commodity of the chosen price normalization. Each of these subsystems may vary with the normalization, although it uses always the same technique, because the normalization commodity and, consequently, the net product of this subsystem may vary with the normalization. Moreover, the normalization commodity and these subsystems may vary with the normalization in such a way that for a given ad uniform rate of profit the ordering of their nominal wage rates changes and so the ordering with respect to their profitability changes. This change in the ordering of the normalization subsystems with respect to their profitability -due to the change in the normalization and in the normalization commodity- appears as if it were a change in the ordering of the given techniques with respect to their profitability.

d) I showed that the ordering -using as a criterion that of the w-r relation- of given techniques with respect to their profitability is actually the ordering - using the same criterion- of the normalization subsystems which correspond to a given normalization with respect to their profitability by showing that the w-r relation of each one of the given techniques is actually the w-r relation of the normalization subsystem which corresponds to a given normalization. Specifically, I showed that for each normalization in each w-r relation:

1. w_{max} is the productivity of labour in price terms of the corresponding normalization subsystem,
2. r_{max} is the maximum rate of profit of the corresponding normalization subsystem,
3. the slope -dw/dr of the w-r relation is always equal to the ratio of the marginal change of the average profit per unit of labour to the marginal change of the rate of profit in the corresponding normalization subsystems multiplied by -1, and
4. that these three magnitudes, in the general case, may vary with the normalization because the normalization commodity i.e. the net product of the normalization subsystem, may change with the normalization and so each normalization subsystem may change.

Nowhere did I claim that the phenomenon of reswitching does not exist, as Kurz/Gehrke reproach me (Kurz/Gehrke, 1994:100). I only claimed and proved that this phenomenon may appear and disappear with the normalization. That is why the impression that Kurz/Gehrke try to create that I supposedly repeat what Levhari (Levhari, 1965) erroneously claimed about the non-existence of the phenomenon, is misleading. Erreygers' reference to Yi's article (Yi, 1980) is equally misleading. The results given in my article have absolutely no relation to

Yi's -correct or incorrect- theses, and consequently, to the answer given to Yi by Ahmad (see Ahmad, 1986).

The critique by Erreygers consists of the following two objections:

1. The appearance and disappearance of the phenomena of switching and reswitching of techniques -due to the change in the normalization-, which I illustrated by means of two numerical examples in Part 6 of my article, is not appearance and disappearance of such phenomena, and
2. The correct criterion for the ordering of decomposable techniques is not the criterion of the w-r relation that I use, but the criterion of cost-minimisation that he uses.

Kurz and Gehrke share these objections and add the objection:

3. that if one normalizes the prices, then the wage rate is determined not only as *a nominal wage rate* -as I claim- but also as a *real* wage rate and that if one gives exogenously the real wage rate, the ordering of techniques does not change with the normalization.

I will respond to the first two objections only since I consider the third to be obviously erroneous[1].

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1. Nevertheless, the following will be useful to us when we will refer to Bidard: The normalization of prices and of nominal wage rates of given techniques does not imply that the composition of the real wage is given and the same with the composition of the normalization commodity and consequently it is not implied that the real wage rate of every technique is given when the prices are normalized and the nominal wage rate is given and it is the same for each technique. That is why the price normalization leaves the composition of the real wage rate of every technique "open". The normalization of prices of given techniques implies only one thing, the following: Because, *firstly,* when in a production system the real wage has the same composition for each profit rate, except the maximum profit rate, then obviously this composition is necessarly the same with that of the net product of the given system and *secondly,* the ordering of given techniques using the criterion of the w-r relation for a given normalization commodity and a given nominal wage rate is the ordering of the corresponding normalization subsystems, i.e. the systems each of which uses one of the given techniques and produces the given normalization commodity as its net product, thus the same net product with all other systems -for these reasons, when the real wage rate has the same composition for each rate of profit and in all techniques, then this composition is necessarily the same with that of the normalization commodity, i.e. of the common net product of the corresponding normalization subsystems.
   But this uniform composition of the real wage rate for each rate of profit and in all tehniques which is the same with the composition of the normalization commodity constitutes only one possibility -the unique- that the normalization allows us to introduce and not a necessity

ad 1:

Erreygers claims that the appearance and disappearance of switch and reswitching points due to the change in the normalization which I showed with the help of two numerical examples in Part 6 of my article, is not in fact the appearance

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imposed on us by the normalization if we consider necessary to introduce in the model a uniform for each rate of profit and in all techniques composition of the real wage rate. The normalization allows but does not impose its introduction.

But the fact that this possibility is unique in the sense that the uniform, for each rate of profit and for all techniques composition of the real wage rate may be the one and only one: the composition of the normalization commodity, implies the following:

If, before the price normalization, I will give exogenously the composition of the real wage rate that holds for each rate of profit and for each technique, then I must normalize prices using as normalization commodity one that has the same composition as the exogenously given composition of the real wage rate because, if I will normalize prices using any other normalization commodity, then the presupposed that holds for each rate of profit and for each technique composition of he real wage rate can not exist, since the only possible composition of the real wage rate that holds for each rate of profit and for each technique is the composition of the normalization commodity. So when one gives exogenously the composition of the real wage rate that holds for each rate of profit and for each technique, the normalization is given and can not vary. Consequently, it is not possible to say, as Kurz/Gerke do, that, when one gives exogenously the composition of the real wage rate, the ordering of techniques does not vary with the normalization, because the normalization can not vary. If one, despite of all these, wants to vary the normalization, then he will find out -if he bothers to inquire- that, contrary to his initial presupposition, according to which the composition of the real wage rate is, for each rate of profit and for all techniques, given and the same, the composition of the real wage rate is not the initially presupposed but it is "open" and that the only composition that holds for each rate of profit and for all techniques is that on which is the same with the composition of the *new* normalization commodity and consequently one different from the initially presupposed.

For the relation between normalization and the composition of the real wage rate the following hold in general: The normalization does not imply anything at all for the composition of the real wage rate, but leaves it "open". But if one, given the normalization, wants to introduce a uniform for each rate of profit and for all techniques composition of the real wage rate, then he can introduce the only one possible: the one that is the same with the composition of the normalization commodity used. And if one wants to normalize prices when he has previously introduced a uniform for all rates of profit and for all techniques composition of the real wage rate, then he can select one and only one normalization commodity, if he does not want to contradict his assumption for the composition of the real wage rate: one with the same composition with the presupposed composition of the real wage rate.

and disappearance of such phenomena. His argument is that we speak of switching (reswitching) when we have a point (two or more points) of intersection of two w-r curves only in the case that the corresponding techniques are neighbouring techniques, i.e. they differ only in one process, whereas the two 2x2 techniques ‘a’ and ‘b’ in each of my two arithmetical examples differ not only in one, but in the two production process. He also observes that in each of the two numerical examples, the techniques are not two, but four because from the combination of the four different production process of the two initially given techniques ‘a’ and ‘b’, arise two more techniques ‘c’ and ‘b’, i.e. a total of four techniques.

Then, he examines my first numerical example, which refers to the phenomenon of switching and shows that between neighbouring techniques only one switch point exists, that between the neighbouring techniques ‘a’ and ‘d’ for r=1/9. But he does not examine if this situation changes with the change of normalization!

He also examines my second numerical example and establishes that a switch point exists between the neighbouring techniques ‘a’ and ‘c’ for r=0,045578, but not reswitching points between neighbouring techniques. However, he does not examine here if the switch point changes with the normalization. He does not also examine if with the change of normalization reswitching appears between neighbouring techniques. Inspite of these, he claims that nothing changes here with the change of normalization!

However, in my article, I examined the question of whether the ordering of any -neighbouring or non-neighbouring- techniques changes with the normalization. And not the question of whether the ordering of only neighbouring techniques changes with the normalization. (The restriction of the interest only to neighbouring techniques has a meaning only in the choice of the more profitable technique and not generally in the ordering of techniques).

Therefore, as a result, I talk about switching (reswitching) when I have a point (two or more points) of intersection of the w-r curves of two techniques both in the case in which the two techniques are neighbouring techniques, and in the case in which they are not neighbouring techniques. So my contention that switching and reswitching appear and disappear with the normalization refers to the general case, i.e. both to the case of neighbouring techniques, and to the case of non-neighbouring techniques.

However, if one distinguishes between neighbouring and non-neighbouring techniques, then the two above mentioned numerical examples confirm my contention only for the non-neighbouring techniques. This is the only point that Erreygers rightfully pinpoints. But this in no way shows, as he thinks, that my contention is erroneous for the neighbouring techniques! Inspite of this, he claims without due examination that the ordering of neighbouring techniques does not change with the normalization[2].

In order to determine whether Erreygers' contention is correct, we accept that "it is true that at switch-points the w-r curves of the techniques in question intersect, but is not true that rates of profits for which the w-r curves of two techniques intersect are always switch-points..., the intersection of w-r curves is a necessary but not a sufficient condition for switching or indifference" (Erreygers 1994:99) and that, as Erreygers obviously implies, the sufficient condition is that the techniques in question are neighbouring. I also accept that when the two w-r curves intersect two or more times, we cannot talk about reswitching but only in the case in which the two techniques are neighbouring techniques. So, I put aside the "misunderstanding" on which, according to Erreygers, I base my whole article (Erreygers 1994:99).

Then, it remains to be shown that Erreyger's claim, according to which neither the ordering of neighbouring techniques changes with the normalization, nor the switch-point and the reswitching-points of neighbouring techniques appear and disappear with the normalization, is erroneous. Obviously, a counter-example suffices. Here is the counter-example:

Let the two neighbouring techniques $[A^{(a)}, \ell^{(a)}]$ and $[A^{(b)}, \ell^{(b)}]$ with

$$A^{(a)} = \begin{bmatrix} 0.5 & 0.25 \\ 0 & 0.75 \end{bmatrix}, \quad \ell^{(a)} = (0.5, 0.5)$$

and

$$A^{(b)} = \begin{bmatrix} 0.5 & 0.25 \\ 0 & 0.749 \end{bmatrix}, \quad \ell^{(b)} = (0.5, 0.252).$$

If we normalize the prices with

$$p_1^a = p_1^b = 1$$

for both techniques 'a' and 'b', we obtain the w-r relation

$$w = 1 - r,$$

with

$$w_{max}^a = w_{max}^b = 1$$

2. The example in Part 5 of my article to which Erreygers refers is an example of neighbouring techniques, the ordering of which changes with the normalization and in such a way that the phenomenon of switch appears and disappears with the change of normalization.

and $$r^a_{max} = r^b_{max} = 1$$

for this normalization both techniques are equally profitable for every r, $0 \le r \le r^a_{max} = r^b_{max} = 1$, and every r, $0 \le r \le 1$, is a point of indifference.

But if we normalize prices with

$$0.25p^a_1 + 0.25p^a_2 = 0.25p^b_1 + 0.25p^b_2 = 1,$$

then the w-r relation of the technique 'a' is

$$w^a = 1 - 3r$$

with

$$w^a_{max} = 1 \text{ and } r^a_{max} = 1/3 = 0.333$$

and the w-r relation of the technique 'b' is

$$w^b = \frac{[1-0.5(1+r)][1-0.749(1+r)]}{0.188-0.062625(1+r)}$$

with

$$w^b_{max} = 1.001 \text{ and } r^b_{max} = 0.335.$$

With this normalization we have only two points of indifference: the points

$$r_1 = 0.00205$$

and

$$r_2 = 0.326145.$$

The ordering of the two given neighbouring techniques did not change with the normalization only for these two values of r. It changed for every other value of r. Also, the maximum relate of profit of the two techniques changed with the normalization! From $r^a_{max} = 1$ and $r^b_{max} = 1$, that they were according to the first normalization they became $r^a_{max} = 1/3$ and $r^b_{max} = 0.335$ with the second normalization. But mainly: with the passage from the second to the first normalization the phenomenon of reswitching that appears with the second normalization disappears.

For these phenomena Erreygers and Kurz/Gehrke do not have any interpretation. But as I have shown in my article, they are explained very easily and simply with the help of the concept of normalization subsystem that Erreygers and Kurz/Gehrke passed over lightly. All these apparent "paradoxes" are consequences of the fact that the w-r relations are not the w-r relations of the techniques, but of the respective normalization subsystems (See Stamatis 1983, Stamatis 1988 and Stamatis 1993).

So both, in the case of the first normalization and in the case of the second normalization, we do not compare and order the two given -the same in both cases- neighbouring techniques 'a' and 'b', but the two different in each of the two cases normalization subsystems: in the first case, two normalization subsystems which both produce the same net product consisting of one unit of the commodity 1 the first one using only the basic part of the technique 'a' and the second one using only the basic part of the 'b' and thus both using the same technique consisting only of the production activity 1, i.e. two identical systems. In the second case two normalization subsystems, which both produce the same net output consisting of 0.25 units of commodity 1 and 0.25 units of commodity 2, the first one using the technique 'a' and the second one using the technique 'b' and thus, two different neighbouring techniques, i.e. two systems which are different both from the above and between them. That is why it is not surprising that the two systems are, in the first case, equivalent for every value of r, and in the second case, equivalent for only two values of r. This change in the ordering comes as a surprise only to those who erroneously think that this ordering refers to the techniques and not to the corresponding normalization subsystems.

ad 2:

Here, Erreygers criticises the conclusions that I draw from the numerical example in Part 5 of my article in which I compare two decomposable neighbouring techniques 'a' and 'b' with

$$A^{(a)} = \begin{bmatrix} 0.5 & 0.25 \\ 0 & 0.75 \end{bmatrix}, \quad \ell^{(a)} = (0.25, 0.25)$$

and

$$A^{(b)} = \begin{bmatrix} 0.5 & 0.25 \\ 0 & 0 \end{bmatrix}, \quad \ell^{(b)} = (0.25, 0.25)$$

The conclusions were the following:

a) If we normalize prices with

$$2p_1^a = 2p_1^b = 1$$

then we obtain the following w-r relation for both techniques 'a' and 'b',

$$w = 1 - r$$

with the consequence according to the criterion of the w-r relation that both techniques are equivalent for each w, $0 \leq w \leq 1$, and so for each r, $0 \leq r \leq 1$.

b) If we normalize prices with

$$0.5p_1^a + 0.5p_2^a = 0.5p_1^b + 0.5p_2^b = 1$$

then we obtain for technique 'a' the w-r relation

$$w^a = 1 - 3r$$

with

$$w_{max}^a = 1 \text{ and } r_{max}^a = 1/3$$

and we obtain for the technique 'b' the w-r relation

$$w^b = \frac{16(1-r)}{7-r}$$

with $w_{max}^b = 16/7$ and $r_{max}^b = 1$,

with the consequence that technique 'b' surpasses technique 'b' for each r, $0 \leq r \leq 1$.

c) Consequently, the ordering of two neighbouring techniques 'a' and 'b' varies with the normalization when the criterion of the w-r relation is used as the ordering criterion.

Erreygers objects to point a) and consequently, to point c). He writes:

"It is easy to show that Stamatis' conclusion for $n' = [2,0]$ is wrong. For this numéraire, technique 'a' determines the following prices and wage:

$$p_1^a = \frac{1}{2},\ p_2^a = \frac{(3-r)}{2(1-3r)},\ w^a = (1-r). \tag{1}$$

This means that we have:

$$p^a B_3 - (1+r)p^a A_3 - w^a I_3 = \frac{3(3-r)(1+r)}{8(1-3r)}. \tag{2}$$

As long as $0 \leq r \leq 1/3$, the right-hand side of (2) is positive or infinite, which means that the third process pays extra profits, i.e. is cheaper than the second. For these values of r, technique 'b' is the only cost-minimising technique. For $r > 1/3$, either p_2^a or w^a is negative, which again means that technique 'a' cannot be an acceptable alternative for technique 'b'. The conclusion is that technique 'a' is *never* the cost-minimising technique" (Erreygers 1994:96).

Before we deal with what Erreygers writes here, we wish to make a more general remark which concerns both Erreygers and Kurz/Gehrke. In order for someone to talk about switch-points and reswitching-points he must, first, normalize uniformly the prices of both techniques in any way but different from that of the normalization equation $w^a = w^b = w = 1$, and, secondly, accept the criterion of the w-r relation as the only correct criterion of comparison and ordering of given techniques.[3] This is what both Erreygers and Kurz/Gehrke do when they talk about switch-points and reswitching-points.

But as soon as they face the apparent paradoxes of the use of the criterion of the w-r relation, which, even if somebody explains to them, they cannot or do not want to comprehend, they resort to the criterion of cost minimisation, deliberately forgetting both that they considered the w-r criterion as self-evidently valid, and that they have to give an self-evidently valid, and that they have to give and explanation of why not long ago they considered the criterion of the w-r relation as valid and now, they consider as valid only the criterion of cost minimisation.

Let us temporarily accept that Erreygers criterion of cost minimisation is the only valid criterion. Under this assumption, we have two corrections to make and two questions to ask.

The first correction is the following: For a given normalization, with normalization commodity n, $n=(2,0)^T$, and for $r=1/3$, the technique 'b' is not the only cost minimising technique. Because the right part of Erreygers' relation (2) is, as the himself points out, indeterminate having as a consequence that the techniques 'a' and 'b' in this case are not comparable using the criterion of cost minimisation.

The second correction is the following: For the normalization with normalization commodity n, $n=(2,0)^T$, and for r, $1/3 < r \leq 1$, *it is not true*, as Erreygers writes, that "either p_2^a or w^a is negative", but, as follows from Erreygers' relation (1), for w^a holds $w^a \geq 0$ and more specifically, $0 \leq w^a < 2/3$, and for p_2^a holds $p_2^a < 0$ and more specifically, $-½ > p_2^a > -\infty$.

And now the questions: Why does Erreygers not compare the two techniques for the given normalization and for r, $1/3 < r < 1$, but observes laconically and, as we have just showed, *erroneously* that "for $r > 1/3$ either p_2^a or w^a is negative"? Because for the given normalization, and for r, $1/3 < r < 1$, the right part of Erreygers' relation (2) is negative and so the technique 'a' surpasses technique 'b' according to his own criterion of cost minimisation! And why he does not, in

3. In this case, one compares for a given w the rate of profits of two techniques or for a given r the nominal wage rate of both techniques. Of course, one is allowed to normalize using $w^a=w^b=w=1$. In this case, one compares for a given r the price vectors of both techniques.

addition, normalize with the normalization commodity n, $n=(0.5, 0.5)^T$, in order to check if the ordering of the two techniques varies with the change of the normalization commodity when we use the criterion of cost minimisation? Because, as we will show immediately, this ordering changes with the change in the normalization commodity!

If we normalize with normalization commodity n, $n=(0.5, 0.5)^T$, then instead of Erreygers' relations (1) and (2), we obtain respectively,

$$p_1^a=\frac{0.25(1-3r)}{1-0.5(1+r)},\ p_2^a=2-p_1^a=2-\frac{0.25(1-3r)}{1-0.5(1+r)},\ w^a=1-3r \tag{1a}$$

and

$$\begin{aligned} &p^aB_3-(1+r)p^aA_3-w^aI_3= \\ &=p_2^a-0.25(1+r)p_1^a-0.25(1-3r)= \\ &=2-\frac{0.25(1-3r)}{1-0.5(1+r)}-\frac{0.625(1+r)(1-3r)}{1-0.5(1+r)}-0.25(1-3r). \end{aligned} \tag{2a}$$

For each r, $0 \leq r \leq 1/3$, the right part of (2a) is positive and consequently technique 'b' surpasses technique 'a'.

Using the above normalization, we obtain

$$w^b=\frac{16(1-r)}{7-r},$$

$$p_1^b=\frac{4(1-r)}{(7-r)[1-0.5(1+r)]},$$

$$p_2^b=2-p_1^b=2-\frac{4(1-r)}{(7-r)[1-0.5(1+r)]}$$

and

$$\begin{aligned} &p^bB_2-(1+r)p^bA_2-w^bI_2= \\ &=p_2^b-(1+r)p_1^b0.25-(1+r)0.75p_2^b-\frac{16(1-r)}{7-r}0.25 \end{aligned} \tag{2b}$$

For each r, $0 \leq r \leq 1$, the right part of (2b) is negative. Consequently, for each r, $0 \leq r \leq 1$, the technique 'b' surpasses technique 'a'.

Therefore, the ordering of techniques 'a' and 'b', using Erreygers' criterion of cost minimisation varies with the normalization.

According to the normalization with normalization commodity n, $n=(2,0)^T$,

- for r, $0 \leq r < 1/3$ technique 'b' surpasses technique 'a',
- for $r = 1/3$, the comparison of the two techniques is impossible, and
- for r, $1/3 < r \leq 1$, technique 'a' surpasses technique 'b'.

According to the normalization with normalization commodity n, $n=(0.5, 0.5)^T$, the technique 'b' surpasses technique 'a' for each r, $0 \leq r < 1/3$. Thus the ordering of two techniques 'a' and 'b', using as criterion Erreygers' cost-minimisation criterion, does not vary with the normalization only for r, $0 \leq r < 1/3$.

Erreygers evades this unpleasant ascertainment, first, because, when we normalize with normalization commodity n, $n=(2,0)^T$, he erroneously claims that, for $r = 1/3$ technique 'b' surpasses technique 'a' (whereas in reality the comparison of the two techniques is impossible) and also avoids comparing the two techniques for r, $1/3 < r \leq 1$, and, secondly, because he does not compare the two techniques using as criterion his own cost minimisation, for the normalization with normalization commodity n, $n=(0.5, 0.5)^T$.

In Footnote 1, page 97, he says:

"The maximum rate of profit, r_{max}, of a technique is defined as the highest rate of profit for which all prices and the wage are non-negative (0 and $+\infty$ are allowed)." This is a strange definition of the maximum rate of profit of a technique. This definition does not bear a new relation to the actual maximum rate of profit of a technique but is subjected to the desire to avoid the problems which arise from the negative prices of commodities, here from the negative price p_2^a of non basic commodity 2 that appears in technique 'a' for r, $1/3 < r \leq 1$, when one normalizes with normalization commodity n, $n=(2,0)^T$.[4]

The fact that the consistent application of Erreygers' criterion of cost minimisation gives for normalization with $n=(2,0)^T$ for r, $1/3 < r \leq 1$, such an unreasonable result does not mean that this criterion is erroneous, but that this result is due to the negative value of p_2^a. This negative price p_2^a is due to the implicit postulation according to which a uniform rate of profit exists for both process of technique 'a'. When we have normalized with normalization commodity $n=(2,0)^T$ and for r, $1/3 < r \leq 1$, then this postulation is fulfilled only for $p_2^a < 0$.

4. The rate of profit of a technique always arises for a given value of w and it is equal to the minimum of the positive or positive and zero values of r that are obtained for a given w. The real maximum rate of profit of a technique is the rate of profit of this technique which is obtained for $w=0$ and cosnequently is equal to the minimum of the positive values of r that are obtained for $w=0$.

But even if we accept as correct the definition of maximum rate of profit that Erreygers gives, meaning in the given case that we have $r^a_{max} = 1/3$ and $r^b_{max} = 1$ when we have normalized prices for both techniques using as normalization commodity the commodity n, $n=(2,0)^T$, and consequently, one affords the comparison of the two techniques for r, $1/3 < r \leq 1$, the following problem remains:

In the case in which we normalize with normalization commodity n, $n=(2,0)^T$ and take $r = 1/3$, the comparison of the two techniques 'a' and 'b' is impossible, while in the case in which we normalize with normalization commodity n, $n=(0.5, 0.5)^T$, and take again $r = 1/3$, the two techniques 'a' and 'b' are comparable and technique 'b' surpasses technique 'a'. So, even if we accept Erreygers' definition of the maximum rate of profit as correct, the ordering of the neighbouring techniques 'a' and 'b', using as criterion Erreygers' cost minimisation criterion, varies for $r = 1/3$ with the normalization in the sense that while for one of the above normalizations the ordering is possible, for the other it is not possible.

This does not mean that Erreygers' criterion of ordering the techniques is erroneous but is a consequence of the fact that for normalization with the normalization commodity n, $n=(2,0)^T$ and for $r = 1/3$ the price p^a_2 of non basic commodity 2 in technique 'a' is in this case indeterminate. The reason for which the price p^a_2 is in this case indeterminate is exactly the same reason for which the price p^a_2 is negative when we normalize prices with normalization commodity n, $n=(2,0)^T$, and take r, $1/3 < r \leq 1$: this is due to the fact that we have implicitly presupposed that a uniform rate of profit exists for both process of the technique 'a' for $r = 1/3$. Obviously, this presupposition is fulfilled only when p^a_2 is indeterminate.

We have seen that the ordering of two decomposable neighbouring techniques of simple production using the criterion of cost minimisation may vary with the normalization commodity. Obviously, this means that the phenomena of switching and reswitching between two decomposable neighbouring techniques of single production -even when they are established by using not the criterion of the w-r relation, but the criterion of cost-minimisation- may appear or disappear or more along the r-axis with changes in the normalization commodity!

But because these "paradoxical" phenomena are only consequences of the change in the ordering of techniques dues to the change of the normalization commodity, the problem is not these "paradoxical" phenomena as such, but the following: Is it possible to have *a linear ordering of techniques* independently of the normalization using as a criterion either the criterion of w-r relation or the criterion of cost minimisation? We will state our views on this, first in the case of neighbouring techniques of single production, and then in the case of neighbouring techniques of joint production, in the form of theses without the corresponding proofs due to the lack of space.

In the case of indecomposable neighbouring techniques of single production, the criteria of w-r relation and of cost minimisation are equivalent. In the same case, the ordering of techniques does not change with changes in the normalization commodity, irrespective of which of the two criteria is used for the ordering. Exactly for this reason, the phenomena of switching and reswitching of techniques that appear are independent of the normalization commodity, i.e. they neither appear and disappear nor more along the r-axis with changes in the normalization commodity.

In the case of decomposable neighbouring techniques of single production, the ordering may vary with changes in the normalization commodity, irrespective of whether the ordering is done with the criterion of the w-r relation or the criterion of the cost minimisation. Here, these two criteria are not in general equivalent but only under one specific condition which, if given, depends exclusively on the normalization commodity.

In the case of the ordering of only two decomposable neighbouring techniques of single production, the two criteria are equivalent if and only if the prices are normalized using as normalization commodity either the commodity v which is produced as gross product by the two versions of the v -the production process in which the two given decomposable neighbouring techniques of single production differ, or a composite commodity which contains either and commodity v or at least one of the commodities in the production of which commodity v enters directly or indirectly in both techniques.

In the case of comparison of more than two decomposable neighbouring techniques of simple production both criteria are equivalent, if and only if, first, the given, more than two decomposable neighbouring techniques of single production differ in only one and the same production process, let it be the v -th, and secondly, the prices have been normalized in the way that we referred to above.

In the last two cases and when the prices have been normalized in the way referred to above and consequently the two criteria are equivalent, we are never led to a paradoxical choice of technique, i.e. in the choice of a profoundly inferior technique - a choice which is possible for other normalization commodities under both criteria.

In the case of comparison of more than two decomposable neighbouring techniques of single production, which differ in more than one production process, i.e. when we have more than one production process in which of them only differs two or more of the given production techniques and accordingly more commodities "v" that are produced by each of these production process, then, first, the two criteria are not equivalent for any normalization commodity and, second, the ordering of the given techniques may vary with the normalization commodity, irrespective of which of the two criteria we use, and moreover, in a way which leads to the choice of profoundly inferior techniques.

In all three of the above mentioned cases of decomposable neighbouring techniques of single production phenomena of switching and reswitching of techniques appear and disappear or more along the r-axis with changes in the normalization commodity.

Erreygers refers us to Bidard (1990). The reference is extremely successful. Christian Bidard in his highly important article mentions that his approach applies only for neighbouring techniques of joint production and not for neighbouring techniques of single production. Nevertheless his approach applies also to decomposable techniques of single production although, as we will see in what follows, in this case Bidard's conditions for linear ordering are relatively strict.

The conditions for linear ordering, which Bidard develops and which we will expound in following, are for indecombosable neighbouring single production techniques superfluous, since these technique always can be linear ordered - otherwise they would hold indiscriminately for every type of techniques. So, whatever follows concerning Bidard's conditions of linear ordering of neighbouring single production techniques refers to decomposable neighbouring single production techniques and to neighbouring joint production techniques.

Bidard's ordering criterion is the criterion of cost minimisation. But he does not, as himself mentions, order according to this criterion for a given rate of profit the given neighbouring techniques of joint production as such, but he orders them for a given profit rate in the form of explicitly determined production systems that use these techniques. We have to do -when they exist- with quasi standard production systems which use these techniques and in each of which the surplus product and the means of production contain, first, all the produced commodities in positive quantities and, second, have the same composition (and consequently, the rate of profit is independent from the prices which are positive but arbitrary and consequently -in contradiction to Bidard's explicit assurance- they do not play any role in the ordering of these systems. Besides, he could not have ordered them according to the criterion of cost minimisation because the two alternative production actives of two neighbouring techniques of joint production do not necessarily produce the same gross product as those of two neighbouring techniques of simple production.

According to Bidard himself, only two neighbouring techniques, i.e. the corresponding two Bidard's quasi standard systems, are not for a given rate of profit always linearly ordered, but they are linearly ordered for a given rate of profit only when, first, their real wages contain all the produced commodities in positive quantities and second, they have the same composition, i.e. because each of Bidard's quasi standard systems uses by assumption exactly one unit of direct labour, when their real wages, first, contact all the produced commodities in positive quantities and, second, they have the same composition.

Bidard's ordering criterion is as we have mentioned, the criterion of cost minimisation. During the application of this criterion Bidard normalizes, as is usually done when this criterion is used, the prices through w=1, i.e. he expresses the prices and the nominal wage rates of the given techniques and, consequently, of the respective quasi standard systems in labour commanded. But at the same time, the criterion of cost minimisation is for a specific normalization of prices which is different from Bidard's normalization w=1, but it does not contradict to it, equivalent to the criterion of the w-r relation. In order to make this equivalence visible we will suppose in the following that Bidard does not normalize prices using w=1, but by putting the price of one commodity equal to a positive constant. But because Bidard orders the neighbouring techniques in the form of those systems that use these techniques which among other have real wages of the same composition, this normalization commodity is necessarily a composite commodity whose composition is the same with the common composition of the real wages of the quasi standard systems which Bidard orders (Compare footnote 1).

When in the case of the ordering of only two neighbouring techniques for the given rate of profit systems exist which, first, use these techniques, second, in each of these the surplus product and the means of production contain all the produced commodities in positive quantities and, thirdly, have real wages which contain all commodities in positive quantities and have the same composition, then these are linearly ordered and, according to Bidard's criterion of cost-minimisation, for a given rate of profit we choose the one with the greater real wage as being superior. If in the same case we normalize prices using as normalization commodity the composite commodity with composition the common composition of real wage rates of the above two systems, then we choose for the given rate of profit with the criterion of w-r relation as the most profitable the one with the greater nominal wage rate which, due to the used normalization, is, at the same time, the one with the greater real wage rate. Consequently, for the used normalization, i.e. the normalization with normalization commodity a composite commodity with composition the common composition of the real wage rate of Bidard's two quasi standard systems, the criterion of cost-minimisation and the criterion of the w-r relation are equivalent. It is obvious that in the case, in which the two given neighbouring techniques are decomposable techniques of simple production, this normalization commodity is not but the normalization commodity which contains and the commodity v about which we have spoken above.[5]

When the given neighbouring techniques are more than two, then Bidard's corresponding quasi standard systems are comparable, according to Bidard himself, only under the same above mentioned condition of the existence of strictly positive real wage rate of the same composition. But the common composition of the strictly positive real wage rates is not uniquely determined, but varies within certain limits. So even when a common composition of the real wage rate exists, the ordering of the above mentioned systems is not uniquely determined, but it is possible to vary with the varying common composition of the real wage rates, i.e. with the normalization commodity. Bidard himself does not leave any doubt about the fact that their ordering is not uniquely determined, but it is possible to vary with the "common direction d", i.e. with this common composition of the real wage rates. Consequently, according to Bidard the ordering of given more than two neighbouring techniques may vary with the normalization commodity. And because it may vary with the normalization commodity -even when this variation preserves the specific composition that Bidard requires, the phenomena of switching and reswitching of techniques, which appear in this case, may appear and disappear or more along the r-axis with the changes in the normalization commodity.

There is only one possibility that the ordering does not vary with the common composition of the real wage rate and, consequently, with the normalization commodity as long as it has, of course, the same composition with the common composition of the real wage rate: when this common composition of the real wage rates of Bidard's quasi standard systems and, consequently, of the normalization commodity, is uniquely determined and consequently cannot vary. When does this happen? Given that in each of Bidard's quasi standard systems the strictly positive surplus and the strictly positive means of production have the same invariant composition, this happens when, first, the real wage rate has in each of these systems the same composition with the surplus product and the means of production and then each of Bidard's quasi standard systems is transformed in a

5. But here, i.e. in the case of two decomposable neighbouring techniques of simple production, Bidard's condition is relatively strict because in this case, the normalization commodity does not need to have the specific composition that Bidard generally requires, but it is sufficient to have the composition which we described above.

quasi corn economy, i.e. in a quasi-one-good-economy, and, second, the real wage rates of all these quasi-one-good-economy, and, second, the real wage rates of all these quasi-one-good-economies have the same composition and, consequently, all of these quasi-one-good-economies produce net products (and gross products) and use means of production of the same composition. In other words: this happens only when Bidard's systems are Sraffian or Vasilakian (see Vassilakis 1983) standard systems with common composition of net products and real wage rates and we select as normalization commodity a commodity having the same composition with the common composition of the net products and the real wage rates of these systems. Because for any other normalization the ordering of these systems may vary with the normalization whether we use the criterion of the w-r relation or the criterion of cost-minimisation.[6] Of course, for each normalization of this type the two criteria are not equivalent.

Under the presupposition, that the two criteria are equivalent and, consequently, the normalization commodity has the composition of the "common direction d", the ordering of the given techniques are uniquely determined only as the ordering of the corresponding to these techniques quasi-one-good-economies which produce net products with the same composition. Of course, such a uniquely determined ordering may exist only for techniques for which Sraffian or Vassilakian standard systems and, consequently, quasi-one-good-economies exist!

Only in this case in which the given neighbouring techniques are ordered in a unique way -under the presupposition of the equivalence of the two ordering criteria and, consequently, of the above mentioned normalization of prices which guarantees this equivalence -the phenomenon of switching of techniques which may appear here they do not appear and disappear nor more along the r-axis with changes in the normalization commodity- for the simple reason that here the normalization commodity cannot vary.[7]

Apart from the trivial case in which all the neighbouring techniques are indecomposable techniques of simple and the above mentioned extremely special case in which for all the given neighbouring techniques quasi-one-good-economies exist which produce net products with the same composition and the normalization commodity has the same composition with the common composition of the net products of these quasi-good-one-economies, only one case of uniquely

6. We have assumed that, in applying this criterion, prices have been normalized in the above mentioned way and not through w=1.

7. Because here the w-r relations of all techniques which are obtained for the normalization in question are obviously linear, then only phenomena of switch, but not of reswitching of techniques may appear.

determined ordering of techniques -not of techniques as such, but again in the form of systems- exists: it is that of von Neumann in which Charasoffian standard systems (see Charasoff 1910 and Stamatis 1988a) are ordered with respect to their profit rate (see Stamatis 1996). But in this ordering wages are included in the capital so that the profit rate is defined as the ratio of the profits to the sum of the price of the means of production and of nominal wages and not as usual as the ratio of profits to the price of the means of production.

It is characteristic that in the cases -apart from the above mentioned trivial case- in which the given techniques are uniquely ordered, and more concretely in the special case of one-good-economies to which we referred and also in the special case of Charasoffian standard systems, the prices of commodities are positive but arbitrary, i.e. they do not play any role in the ordering. And a contratio: in cases in which -apart from the above mentioned trivial case, the prices play a role in the ordering, i.e. in all cases of decomposable neighbouring techniques of single production and in all cases of neighbouring techniques of joint production apart from the two above mentioned special cases, the ordering of given neighbouring techniques is not uniquely determined but depends on the normalization commodity. The latter means that the money introduced in the model through the normalization-, i.e. through the only possible way of its introduction - is not "neutral" since the measured in this money magnitudes depend on the way that it is introduced, i.e. from the normalization commodity and, consequently, that it does not "represent" real money.

In reality my two papers answer to the following important questions: What does the w-r-relation express? Why a variation of the price normalization equation may change the location of the point ($w=0$, $r=r_{max}$) of the w-r-relation and it may change positive commodity prices to zero and indeterminate commodity prices to positive ones? Why a variation of the price normalization equation may change the ordering of given techniques not only according to the criterion of the w-r-relation but also according to the criterion of the cost minimization? When does the ordering of given techniques according to the criterion of the w-r-relation and according to the criterion of cost minimization lead to the same results, i.e. when those criteria are equivalent?

We answered these questions with the help of the concepts of normalization commodity and normalization subsystem, showing that the w-r-relation of each technique is actually the w-r-relation of the corresponding normalization subsystem and that the comparison of techniques according to the criterion of the w-r-relation as well as with that of cost minimization is actually a comparison of the corresponding normalization subsystems.

In their criticism Erreygers and Kurz/Gehrke have neither posed nor answered any of these questions. Without any justification, they hold against our proof, in consequence of which the ordering of techniques according to the criterion of the w-r-relation may changes with the price normalization, that the only admissible criterion is that of the cost minimization. Thereby, they do not inquire the relation between the above criterion and that of the w-r-relation and they ignore that also the ordering of techniques may change with the price normalization.

In conclusion, we have shown in both our papers that the traditional approach to the neorricardian theory of prices, income distribution and comparison of techniques leads to paradoxes as the case of decomposable techniques reveals. We have explained these paradoxes by using the concepts of the normalization commodity and the normalization subsystem. Erreygers and Kurz/Gehrke do not want to be confronted with them and they think that they exorcise them by their refusal to recognise their existence, and consequently they consider that every attempt to explain them is either superfluous or misguided and incorrect.

References

Ahmad, S. (1986), Sraffa Numéraire and Reswitching: A Note. *Journal of Macroeconomics*, vol. 8.

Bidard, Ch. (1990), An Algorithmic Theory of the Choice of Techniques. *Econometrica*, vol. 58.

Bidard, Ch. (1991), *Prix, Reproduction, Rareté,* Dunod.

Charasoff, G. (1910), *Das System des Marxismus. Darstellung und Kritik*, Verlag Hans Bondy, Berlin.

Erreygers, G. (1994), The Impossibility of a Comparison of Techniques: a Comment. *Jahrbücher für Nationalökonomie und Statistik*, Bd. 213.

Kurz, H.D./Gehrke, Ch. (1994), On the Choice of Techniques: A Comment. *Jahrbücher für Nationalökonomie und Statistik*, Bd. 213.

Levhari, D. (1965), A Nonsubsitution Theorem and Reswitching of Techniques, *Quarterly Journal of Economics*, vol. 79.

Neumann, J. von (1938), Über ein ökonomisches Gleichungssystem und eine Verallgemeinerung des Brouwerschen Fixpunktsatzes, in: K. Menger (Hrsg.), *Ergebnisse eines mathematischen Kolloquiuns*, Wien.

Pasinetti, L.L. (1977), *Lectures on the Theory of Production,* Columbia University Press, New York.

Stamatis, G. (1983), *Sraffa und sein Verhältnis zu Ricardo und Marx,* Göttingen.

Stamatis, G. (1983a), Probleme einer fehlenden Wertformanalyse. *Schriftenreihe Mehrwert*, Nr. 25.

Stamatis, G. (1988), *Über das Normawaresubsystem und die w-r-Relation. Ein Beitrag zur Theorie linearer Produktionsysteme.* Verlag Kritiki, Athens.

Stamatis, G. (1988a), Georg Charasoff - a pioneer in the theory of linear production systems, mimeo, Athens.

Stamatis, G. (1993), The Impossibility of a Comparison of Techniques and of the Ascertainement of a Reswitching Phenomenon. *Jahrbücher für Nationalökonomie und Statistik*, vol. 211.

Stamatis, G. (1996), The von Neumann Model of General Equilibrium, mimeo, Athens.

Yi, G.J.L. (1982), The Choice of Numéraire, Capital Intensity Uniqueness, and the Reswitching of Techniques, *Journal of Macroeconomics*, vol. 4.

Vassilakis, S. (1983), Non-basics, the Standard Commodity, and the Uniformity of the Profit Rate, in: Stamatis 1983.

κυκλοφορεί

ΧΡΗΜΑ
ΠΙΣΤΗ
ΤΡΑΠΕΖΕΣ

Είναι γεγονός αναμφισβήτητο ότι το χρήμα ασκεί σημαντικό ρόλο στις σύγχρονες οικονομίες.

Το χρήμα θεωρείται μια από τις σπουδαιότερες ανακαλύψεις του ανθρώπου, ίσως η τρίτη κατά σειρά μετά την ανακάλυψη της φωτιάς και του τροχού και διαδραματίζει ουσιαστικό ρόλο στη ζωή των ανθρώπων.

Οι κοινωνίες θα παρέμεναν στο στάδιο της υπανάπτυξης, ίσως και στο πρωτόγονο στάδιο, χωρίς την ανακάλυψη του χρήματος.

Για την εμφάνιση και ιστορική εξέλιξη του χρήματος μπορούμε να πούμε ότι δεν γνωρίζουμε αρκετά ακόμη, παρόλο που υπάρχει μεγάλος αριθμός ερευνητικών εργασιών πάνω στο θέμα αυτό.

Είναι σίγουρο πάντως ότι, το χρήμα δεν ανακαλύφθηκε σε κάποια συγκεκριμένη στιγμή και σε κάποια συγκεκριμένη χώρα με τη μορφή που έχει σήμερα, αλλά πέρασε από πολλά στάδια εξέλιξης προτού φτάσει στη σημερινή μορφή του.

Ο Αριστοτέλης (384-322 π.Χ.) διατύπωσε την άποψη ότι οι άνθρωποι επινόησαν το χρήμα για λόγους σκοπιμότητας, δηλαδή για τη διευκόλυνση διεκπεραίωσης των συναλλαγών τους.

Την άποψη αυτή υποστήριξαν πολλοί οικονομολόγοι, όπως π.χ. ο A. Smith, ο F.Y. Wisser, κ.ά.

ΕΛΛΗΝΙΚΑ ΓΡΑΜΜΑΤΑ

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